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Numerical solution of nonlinear weakly singular partial integro-differential equation via operational matrices

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ABSTRACT

In this paper, we propose and analyze an efficient matrix method based on shifted Legendre polynomials for the solution of non-linear volterra singular partial integro-differential equations(PIDEs). The operational matrices of integration, differentiation and product are used to reduce the solution of volterra singular PIDEs to the system of non-linear algebraic equations. Some useful results concerning the convergence and error estimates associated to the suggested scheme are presented. Illustrative examples are provided to show the effectiveness and accuracy of proposed numerical method.

1. Introduction

Volterra type integral equations with weakly singular kernel appears in many problems of mathematical physics and chemical reaction, such as theory of elasticity, hydrodynamics, heat conduction, stereology [1], the radiation of heat from a semi infinite solids [2] and many other applications. Equations of this type have been studied by several authors [illegible].

In this paper we study second kind Volterra singular PIDE of the form

$$\phi_t = \phi(x,t) + g(x,t) + \int_0^x \int_0^t \frac{G\phi(\zeta,\eta)}{(x-\zeta)^{\alpha}}\, d\eta\, d\zeta \qquad \begin{array}{r} 0 \le \zeta \le x,\ 0 \le \eta \le t \\ (x,y) \in [0,1] \times [0,1] \\ 0 < \alpha < 1 \end{array} \tag{1}$$

with initial condition $\phi(x,0) = \phi_0(x)$.

Where, ϕ is unknown function in $\Lambda(=[0,1]\times[0,1])$ which should be determined and the functions g, $\phi_0(x)$ are known. G is a non-linear operator. The functions $g(x,t)$ and $\phi(x,t)$ are assumed to be sufficiently smooth in order to guarantee the existence and uniqueness of a solution $\phi \in C(\Lambda)$. We assume that the non-linear term $G\phi$ satisfies the Lipschitz condition in $L^2(\Lambda)$. It is clear that in the above equation the kernel function has a weak singularity at the origin.

Problems involving PIDEs arise in fluid dynamics, engineering, mathematical biology and other areas. In general, PIDEs are difficult to solve analytically. Main challenges in solving PIDEs analytically are due to many factors, such as non-linearity, non-local phenomena, multi-dimensionality. Various numerical techniques have been developed for the solution of PIDEs

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