Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Operational matrix approach for the solution of partial integro-differential equation

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ARTICLE INFO

Keywords:
Two dimensional singular partial integro-differential equation
2D shifted Legendre polynomial
Operational matrix of differentiation
Almost operational matrix of integration

ABSTRACT

In this paper, an effective numerical method is introduced for the treatment of Volterra singular partial integro-differential equations. They are based on the operational and almost operational matrix of integration and differentiation of 2D shifted Legendre polynomials. The methods convert the singular partial integro-differential equation in to a system of algebraic equations. Convergence analysis and error estimates are derived for the proposed method. Illustrative examples are included to demonstrate the validity and applicability of the technique.

1. Introduction

Volterra type integral equations arise in a variety of science and technology fields [1]. An area of increasing scientific interest over the past decades is the study of Volterra integro-differential equation. The theory and application of Volterra integro-differential equations play an important role in the mathematical modeling of many fields: physical phenomena, biological model, chemical kinetics, engineering science and application in heat flow [12]. Several numerical methods for approximating the solution of Volterra integral equations with weakly singular kernel are known [[illegible]]. Also various numerical techniques have been developed for the solution of partial integro-differential equations, see for examples [11,14,15,17–19].

The main aim of this paper is to study second kind Volterra singular partial integro-differential equation of the form

$$u_y = u(x,y) + f(x,y) + \int_0^x \int_0^y \frac{G(u(s,t))}{(x-s)^\alpha}\, dt\, ds \quad 0 \le s \le x, \quad 0 \le t \le y$$
$$(x,y) \in [0,1] \times [0,1]$$
$$0 < \alpha < 1 \tag{1}$$

with initial condition $u(x,0) = u_0(x)$.

Where, u is unknown function in $\Omega (= [0,1] \times [0,1])$ which should be determined and the functions f, $u_0(x)$ are known. G is a linear or non-linear differential operator. It can be seen that in above equation the kernel function has a singularity at the origin. The functions $f(x,y)$ and $u(x,y)$ are assumed to be sufficiently smooth in order to guarantee the existence and uniqueness of a solution $u \in C([0,1] \times [0,1])$. Equations of the form (1) are usually difficult to solve analytically so it is required to obtain an efficient approximation or numerical methods.

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