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Two-dimensional shifted Legendre polynomial collocation method for electromagnetic waves in dielectric media via almost operational matrices

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In this paper, a numerical solution of fractional partial differential equations (FPDEs) for electromagnetic waves in dielectric media will be discussed. For the solution of FPDEs, we developed a numerical collocation method using an algorithm based on two-dimensional shifted Legendre polynomials approximation, which is proposed for electromagnetic waves in dielectric media. By implementing the partial Riemann–Liouville fractional derivative operators, two-dimensional shifted Legendre polynomials approximation and its operational matrix along with collocation method are used to convert FPDEs first into weakly singular fractional partial integro-differential equations and then converted weakly singular fractional partial integro-differential equations into system of algebraic equation. Some results concerning the convergence analysis and error analysis are obtained. Illustrative examples are included to demonstrate the validity and applicability of the technique. Copyright © 2017 John Wiley & Sons, Ltd.

Keywords: two-dimensional shifted Legendre polynomials; FPDEs; FPIDEs; partial Riemann–Liouville fractional derivative; collocation method; operational matrix of differentiation and integration

1. Introduction

We have known that the dielectric relaxation in solids, which is described by the complex frequency dependent dielectric sensitivity $\chi^*(\omega) = \chi'(\omega) - i\chi''(\omega)$, establish the universal power law dependence:

$$\chi'(0) - \chi'(\omega) \sim \omega^{m}, \chi''(\omega) \sim \omega^{m}, \omega << \omega_p, \tag{1}$$

$$\chi'(0) \sim \omega^{n-1}, \chi''(\omega) \sim \omega^{n-1}, \omega >> \omega_p, \tag{2}$$

where $\chi'(0)$ is the static polarization, $0 < n, m < 1$, and ω_p is the loss peak frequency.

As a result, we obtained fractional partial differential equations (FPDEs) for electromagnetic waves in dielectric media (EWDM) using the Maxwell equation. Such a power law dependence in the frequency domain results in the connection between the electric field **E** and the polarization density **B** expressed as a weakly singular Liouville integral, and as a result, the field equations take a form of FPDEs [1]:

$$\left({}_0D_t^{\alpha}\mathbf{E}\right)(t,r) - \lambda_1\left({}_0D_t^{\beta}\mathbf{E}\right)(t,r) + \lambda_2\left(\text{grad div}\mathbf{E}(t,r) - \nabla^2\mathbf{E}(t,r)\right) = -\mu\lambda_2\frac{\partial j(t,r)}{\partial t} \tag{3}$$

$$\left({}_0D_t^{\alpha}\mathbf{B}\right)(t,r) - \lambda_1\left({}_0D_t^{\beta}\mathbf{B}\right)(t,r) - \lambda_2\nabla^2\mathbf{B}(t,r) = \mu\lambda_2\text{curl}j(t,r). \tag{4}$$

Here, the constant coefficients λ_1 and λ_2 depend on the frequency independent properties of a medium, and μ is the magnetic constant, $1 \leq \beta < \alpha < 3$. Note that such a form allows simultaneous consideration of both systems, before and after the peak frequency and the conversion between them. The last aspect can be perspectively important because of its generality for the consideration of non-trivial transitions, which occur in modern composite materials formed by ferroelectric nanoparticles in a polymer matrix [2].

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