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Application of Bernoulli matrix method for solving two-dimensional hyperbolic telegraph equations with Dirichlet boundary conditions

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ABSTRACT

The present article is devoted to develop a new approach and methodology to find the approximate solution of second order two-dimensional telegraph equations with the Dirichlet boundary conditions. We first transform the telegraph equations into equivalent partial integro-differential equations (PIDEs) which contain both initial and boundary conditions and therefore can be solved numerically in a more appropriate manner. Operational matrices of integration and differentiation of Bernoulli polynomials together with the completeness of these polynomials are used to reduce the PIDEs into the associated algebraic generalized Sylvester equations which can be solved by an efficient Krylov subspace iterative (i.e., BICGSTAB) method. The efficiency of the proposed method has been confirmed with several test examples and it is clear that the results are acceptable and found to be in good agreement with exact solutions. We have compared the numerical results of the proposed method with radial basis function method and differential quadrature method. Also, the method is simple, efficient and produces very accurate numerical results in considerably small number of basis functions and hence reduces computational effort. Moreover, the technique is easy to apply for multidimensional problems.

1. Introduction

Partial differential equations (PDEs) are of widespread interest because of their connection with phenomena in the physical world. For, instance, principle of mass balance in a known physical domain or traffic flow on a highway can be modeled in terms of PDEs [1,2]. Also, PDEs can be used to model some principle events in large IP networks [3]. Most of the physical problems like heat conduction, wave propagation, laser beam models are modeled as PDEs. Moreover, they are widely used for modeling and simulation of various complex phenomena in science and engineering such as migration of the contaminants in a stream, smoke plume in atmosphere, tracer dispersion in porous medium etc.

Among these PDEs, the hyperbolic PDEs (HPDEs) have considerable importance in science and engineering. HPDEs have significant role in formulating fundamental equations in atomic physics [4] and are also very useful in understanding various physical phenomena in applied sciences. In recent years, it is found that much efforts have been taken for the numerical solution of the hyperbolic telegraph equations due to their wide applications. Telegraph equations are commonly used in the study of wave propagation of electric signals [5] and also in wave phenomena [6]. Since, in most cases it is difficult and also

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