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Application of wavelet collocation method for hyperbolic partial differential equations via matrices

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ABSTRACT

In this work, we developed an efficient computational method based on Legendre and Chebyshev wavelets to find an approximate solution of one dimensional hyperbolic partial differential equations (HPDEs) with the given initial conditions. The operational matrices of integration for Legendre and Chebyshev wavelets are derived and utilized to transform the given PDE into the linear system of equations by combining collocation method. Convergence analysis and error estimation associated to the presented idea are also investigated under several mild conditions. Numerical experiments confirm that the proposed method has good accuracy and efficiency. Moreover, the use of Legendre and Chebyshev wavelets are found to be accurate, simple and fast.

1. Introduction

Wavelet theory, developed mostly over the last 30 years, has generated a tremendous interest in many areas of research in sciences and engineering [1–6]. However, most of the applications of wavelets have been focused on analyzing the data and using the wavelets as a tool for data compression [7,8]. In recent years, solving partial differential equations (PDEs) with wavelets has received considerable attention among many researchers. Most of the physical problems like heat conduction, wave propagation, laser beam models are modeled as PDEs whose solutions cannot be easily obtained by the classical methods. This may be due to either the nonlinearity associated with the equation or inappropriate solution space since PDEs are usually applied for simulating the physical phenomena in many branches of sciences and engineering. Therefore the solution of PDEs should be considered in the best manner for extracting the behavior of unknown variables that are formulated under the considerable models. On the other hand, only for some special classes of PDEs analytical solutions are available and in many cases it is impossible to obtain the analytical solutions. Therefore, a wide class of numerical methods were introduced such as spectral method [9], finite difference methods (FDMs), finite element methods (FEMs) (for instance see [10] and the references therein). If the solution of physical problem has regular features, any of these numerical techniques can be applied. However, in many physical problems there exists a multiplicity of very different spatial and temporal scales in the solution, as in strongly time-dependent non-Newtonian convection, formation of shock waves in compressible gas flow, pattern formation in hydrodynamics systems and turbulent flow around bluff bodies. This particular attribute of multiple spatial scales, which possibly change over time will put great strain on these numerical methods. Spectral methods have problems capturing large irregularities of the solution. The main difficulties of existing adaptive finite difference methods or finite element methods is developing computationally efficient robust adaptive procedure, which dynamically adapts the

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