Physica A 541 (2020) 123784

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

An efficient matrix approach for two-dimensional diffusion and telegraph equations with Dirichlet boundary conditions

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ARTICLE INFO

Article history:
Received 12 May 2019
Received in revised form 11 November 2019
Available online 19 December 2019

Keywords:
Two-dimensional diffusion equation
Two-dimensional telegraph equation
Euler polynomials
Operational matrices
Euler matrix method

ABSTRACT

This article provides an efficient matrix approach by using Euler approximation for solving numerically the two-dimensional diffusion and telegraph equations subject to the Dirichlet boundary conditions. First, the main equation is reduced into partial integro-differential equations (PIDEs) and then operational matrices of differentiation and integration of Euler polynomials transform these PIDEs into algebraic generalized Sylvester equations. The inclusion of several test examples confirms the predicted accuracy and effectiveness of the method. Comparison of obtained numerical results is made with some earlier works (Zogheib and Tohidi, 2016, Singh et al., 2018).

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1. Introduction

Partial differential equations (PDEs) furnish a vicenary description for many central models in physical, biological, and social sciences [1–3]. Due to their key role in several areas of applied sciences, PDEs are studied extensively by many researchers. In this article, we examine the feasibility of applying Euler matrix method for the following two-dimensional diffusion equation [4]:

$$\frac{\partial \omega}{\partial \tau} = \frac{\partial^2 \omega}{\partial \xi^2} + \frac{\partial^2 \omega}{\partial \eta^2} + f(\xi, \eta, \tau), \quad 0 < \xi, \eta < 1, \ 0 < \tau \leq 1, \tag{1.1}$$

with the following initial condition

$$\omega(\xi, \eta, 0) = \kappa(\xi, \eta), \quad 0 \leq \xi, \eta \leq 1, \tag{1.2}$$

and the Dirichlet boundary conditions

$$\begin{cases} \omega(0, \eta, \tau) = f_1(\eta, \tau), \ \omega(1, \eta, \tau) = f_2(\eta, \tau), \quad 0 \leq \eta \leq 1, \ 0 < \tau \leq 1 \\ \omega(\xi, 0, \tau) = g_1(\xi, \tau), \ \omega(\xi, 1, \tau) = g_2(\xi, \tau), \quad 0 < \xi < 1, \ 0 < \tau \leq 1. \end{cases} \tag{1.3}$$

We assume that ω and f are smooth enough.

Among PDEs, two-dimensional diffusion equations are of special interest because of their wide applications in physical and applied sciences. They have been used to model chemical exchange reactions, the transport of ground water in

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https://doi.org/10.1016/j.physa.2019.123784
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