

## Convergence of modified Ishikawa's iteration process for asymptotically pseudocontractive mappings

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**Abstract.** We establish a common fixed point theorem for a finite family of continuous uniformly  $L$ -Lipschitzian asymptotically pseudocontractive mappings to prove the strong convergence of modified Ishikawa's method provided that the interior of the set of common fixed points is nonempty, wherein the compactness assumption is not imposed either on the mappings or on the space. Moreover, the computation of closed convex set for each iteration is not required. The results obtained in this paper are improvements over many results that have been proved for this class of nonlinear mappings.

### 1. Introduction

Let  $C$  be a nonempty subset of a real Hilbert space  $H$ . A mapping  $T : C \rightarrow H$  is called

1. nonexpansive, if

$$\|Tx - Ty\| \leq \|x - y\|, \quad \forall x, y \in C,$$

2. asymptotically nonexpansive [5], if for each  $n \in \mathbf{N}$ , there exists a sequence  $\{k_n\} \subset [1, \infty)$  with  $\lim_{n \rightarrow \infty} k_n = 1$  such that

$$\|T^n x - T^n y\| \leq k_n \|x - y\|, \quad \forall x, y \in C,$$

3. uniformly  $L$ -Lipschitzian, if for each  $n \in \mathbf{N}$ , there exists a positive constant  $L$  such that

$$\|T^n x - T^n y\| \leq L \|x - y\|, \quad \forall x, y \in C,$$

4. pseudocontractive, if

$$\langle Tx - Ty, x - y \rangle \leq \|x - y\|^2, \quad \forall x, y \in C,$$

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