Math Sci (2014) 8:131–138
DOI 10.1007/s40096-015-0138-9

ORIGINAL RESEARCH

Shorter proofs of some recent even-tupled coincidence theorems for weak contractions in ordered metric spaces

Anupam Sharma · Mohammad Imdad · Aftab Alam

Received: 23 August 2014 / Accepted: 9 January 2015 / Published online: 10 February 2015

Abstract In this paper, we prove some recent even coincidence theorems due to Imdad et al. (Bull Math Anal Appl 5(4): 19-39, 2013) using a method of reduction from the respective coincidence theorems for mappings with one variable in ordered complete metric spaces. Our technique of proof is different, slightly simpler, shorter and more effective than the ones used in Imdad et al.

Keywords Partially ordered set · Compatible mapping · Mixed g-monotone property · n-tupled coincidence point · n-tupled fixed point

Mathematics Subject Classification 54H10 · 54H25

Introduction

The investigation of fixed points in ordered metric spaces is a relatively new development which appears to have its origin in the paper of Ran and Reurings [30] which was well complimented by Nieto and López [25]. Ran and Reurings' fixed point theorem extended and refined by many authors, (for details see [8, 12, 24–27, 37]).

The concept of coupled fixed point was introduced by Guo and Lakshmikantham [11]. In [5], Bhaskar and Lakshmikantham introduced the notion of mixed monotone property for a mapping $F : X^2 \to X$ and proved some coupled fixed point theorems for weakly linear contractions enjoying mixed monotone property in ordered complete metric spaces. In this continuation, Lakshmikantham and Ćirić [22] generalized these results for nonlinear contraction mappings by introducing two ideas namely: coupled coincidence point and mixed g-monotone property. In an attempt to extend the definition from X^2 to X^3, Berinde and Borcut [4] introduced the concept of tripled fixed point and utilize the same to prove some tripled fixed point theorems. After that, Karapınar [16] introduced the quadrupled fixed point to prove some quadrupled fixed point theorems for nonlinear contraction mappings satisfying mixed g-monotone property (for more details see [17, 18]). Recently, Samet and Vetro [32] extended the idea of coupled as well as quadrupled fixed point to higher dimensions by introducing the notion of fixed point of n-order (or n-tupled fixed point, where $n \in \mathbb{N}$ and $n \geq 3$) and presented some n-tupled fixed point results in complete metric spaces, using a new concept of f-invariant set. Here it can be pointed out that the notion of tripled fixed point due to Berinde and Borcut [4] is different from the one defined by Samet and Vetro [32] for $n = 3$ in the case of ordered metric spaces in order to keep the mixed monotone property working. Recently, Imdad et al. [13] extended the idea of mixed g-monotone property to the mapping $F : X^n \to X$ (where n is even natural number)and proved an even-tupled coincidence point theorem for nonlinear contraction mappings satisfying mixed g-monotone property. Basically their results are true for only even n but not for odd ones (for details see [15]). Further, Imdad et al. [14] proved some even-tupled coincidence theorems under nonlinear weak contractions due to Choudhury et al. [9].

A. Sharma (✉) · M. Imdad · A. Alam
Department of Mathematics, Aligarh Muslim University, Aligarh 202 002, India
e-mail: anmusharma241@gmail.com

M. Imdad
e-mail: mhimdad@yahoo.co.in

A. Alam
e-mail: aafu.amu@gmail.com