Applied Mathematics E-Notes, 19(2019), 689-707 © ISSN 1607-2510
Available free at mirror sites of http://www.math.nthu.edu.tw/∼amen/

On The Convergence Of A New Proximal Point Algorithm Of Generalized Nonexpansive Mappings In CAT(0) Spaces*

Anupam Sharma†

November 13, 2019

Abstract

In this paper, we introduce a new proximal point three step algorithm to establish some strong and Δ-convergence theorems under some suitable conditions and approximate the common fixed point of two finite families of generalized nonexpansive mappings in $CAT(0)$ spaces. Our results generalize and improve several previously known results of the existing literature.

1 Introduction

Let K be a nonempty subset of a metric space (X, d). A mapping $T : K \to K$ is said to be nonexpansive if

$$d(Tx, Ty) \leq d(x, y) \quad \forall\ x, y \in K.$$

An element $x \in K$ is said to be a fixed point of T if $Tx = x$. The set of all fixed points of T is denoted by by $F(T)$.

Suzuki [40] introduced a generalization of nonexpansive maps and referred them as maps satisfying condition (C) and also established some fixed point theorems for these maps. A mapping $T : K \to K$ is said to satisfy condition (C) if

$$\frac{1}{2}d(x, Tx) \leq d(x, y) \Rightarrow d(Tx, Ty) \leq d(x, y),\ \forall\ \ x, y \in K.$$

Every nonexpansive mapping satisfies condition (C) on K. Some of the examples of noncontinuous mappings satisfying condition (C) are mentioned in [40].

Recently, García-Falsat *et al.* [16] defined two new generalizations of condition (C) and termed them as condition (E) and condition (C_λ). They also studied the existence of fixed points and asymptotic behavior under these conditions.

A mapping $T : K \to K$ is said to satisfy condition (C_λ) if for all $x, y \in K$ and $\lambda \in (0, 1)$,

$$\lambda d(x, Tx) \leq d(x, y) \Rightarrow d(Tx, Ty) \leq d(x, y).$$

*Mathematics Subject Classifications: 47H05, 47H09, 47H10.

†Department of Mathematics and Statistics, Indian Institute of Technology, Kanpur 208 016, India