

## SOME COMMON FIXED POINT THEOREMS IN METRIC SPACES UNDER A DIFFERENT SET OF CONDITIONS

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**Abstract.** The purpose of this paper is to prove some new common fixed point theorems in metric spaces for weakly compatible mapping satisfying an implicit relation under a different set of conditions, which unify and generalize most of the existing relevant fixed point theorems. While proving our results, we utilize an implicit function due to Popa et al. [Using implicit relations to prove unified fixed point theorems in metric and 2-metric spaces. Bull. Malays. Math. Sci. Soc. (2) 33 (1) (2010), 105-120] keeping in view their unifying power. Our results improve some recent results contained in Imdad and Ali [Jungck's common fixed point theorem and (E.A) property. Acta Math. Sinica, Eng. Ser. 24(1) (2008), 87-94]. Some related results and illustrative examples to highlight the realized improvements are also furnished.

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### 1. Introduction and Preliminaries

In nonlinear functional analysis, fixed point theory is indispensable due to its wide application to nonlinear sciences besides various research fields in mathematics. One of the essential and initial result in this direction was proved by Stefan Banach [3] in 1922. The classical results of Banach [3] and Edelstein [11] continue to be the source of inspiration for many researchers working in the area of metric fixed point theory. A metrical common fixed point theorem generally involves conditions on commutativity, continuity, completeness and suitable containment of ranges of the involved mappings besides an appropriate contraction condition and researchers in this domain are aimed at weakening one or more of these conditions.

In 1976, Jungck [22] generalized the Banach contraction principle by using the notion of commuting mappings and settled the open problem that a pair of commuting and continuous self-mappings on the unit interval  $[0, 1]$  need not have a common fixed point [4, 13]. He also generalized the idea of weakly commuting mappings due to Sessa [42] and showed that the compatible pair

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