

Fixed point theorems for \mathcal{L} -fuzzy mappings in \mathcal{L} -fuzzy metric spaces

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Abstract. In this paper, we prove fixed points for hybrid pairs of \mathcal{L} -fuzzy and non-self mappings in non-Archimedean \mathcal{L} -fuzzy metric spaces. First, we define the notion of joint common limit range property ((JCLR) property) in such spaces. Further, we prove some common fixed point theorems for \mathcal{L} -fuzzy and non-self mappings in non-Archimedean \mathcal{L} -fuzzy metric spaces. Also, an integral contractive condition and implicit relation have been used in some results. These theorems extend, generalize and improve corresponding previous results in metric, fuzzy metric and \mathcal{L} -fuzzy metric spaces.

Keywords: \mathcal{L} -Fuzzy metric spaces, \mathcal{L} -fuzzy mappings, (JCLR) property, common fixed point.

1. Introduction

In 1967, Goguen [12] introduced the notion of \mathcal{L} -Fuzzy sets as a generalization of fuzzy sets [33]. In 2007, Saadati et al. [24] introduced the concept of \mathcal{L} -fuzzy metric spaces for arbitrary t-norms as an extension of fuzzy metric spaces [13, 17]. Also, in 2007, Sedghi and Shobe [28] proved a common fixed point theorem for two pairs of self mappings (with another conditions on mappings such as compatibility, weakly compatibility and continuity) in complete \mathcal{L} -fuzzy metric spaces using a class of implicit relation. In 2008, Saadati et al. [25] extended the notion of weakly commutativity to ψ -weakly commutativity in \mathcal{L} -fuzzy metric spaces and proved a common fixed point theorem for two ψ -weakly commuting self mappings in a complete \mathcal{L} -fuzzy metric spaces.

In 2009, Saadati and Mansour [23] defined the notion of non-Archimedean \mathcal{L} -fuzzy metric spaces.

Further, there exist some fixed points and another applications in \mathcal{L} -fuzzy metric spaces and \mathcal{L} -fuzzy normed spaces in [15, 27, 31, 32] and others.

In 2011, Sintunavarat and Kumam [29] introduced the notion of common limit range property (or (CLR) property) for a pair of mappings. Further, in 2014, Ahmed and Nafadi [2] extended this notion for single valued and multi valued mappings in fuzzy metric spaces. Also, Chauhan et al. [7] introduced the concept of (JCLR) property. Recently, Rashid et al. [19] established the existence of common \mathcal{L} -fuzzy fixed point theorem in complete metric spaces.

In this paper, we define the notion of (JCLR) property for hybrid pairs of \mathcal{L} -fuzzy and non-self mappings in non-Archimedean \mathcal{L} -fuzzy metric spaces, i.e., the results of Heilpern [11] are extended to \mathcal{L} -fuzzy analogue in non-Archimedean \mathcal{L} -fuzzy metric spaces. Further, we prove fixed point theorems by using implicit relation and integral contractive condition. These theorems generalize, extend and improve the corresponding previous results in [1–3, 5, 16, 18] and others.

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