Design of Second order Differentiator using Micro-strip lines

Narendra Mohan Mishra
Prasar Bharti Doordarshan Kendra, Gorakhpur, Uttar Pradesh, India
narendra.dtu@gmail.com

Priyanka Jain
Department of ECE
Delhi Technological University, New Delhi, India
priyajain2000@rediffmail.com

Narbada Prasad Gupta
Department of ECE
Ideal Institute of Technology, Ghaziabad, India
ernarbada@gmail.com

Pramila
Prasar Bharti, Earth Station
Todapur, New Delhi, India
pramila.dtu@gmail.com

***Abstract* — Novel differentiator design technique is discussed by means of comparison of transfer function of differentiator define and develop in the Digital signal Processing (DSP) with the transfer function develop by cascade connection of transmission-lines configuration. Z-parameter is then used to define the scattering characteristics of equal electrical length transmission line. Finally, Cascade configuration of transmission-lines will develop second order differentiator and verified it by simulation result. Simulation result validates the mathematical observation obtained.**

***Keywords*—Equal-length line, Chain scattering parameters, micro-strip line, Z-transform.**

I. Introduction

Differentiation is a mathematical tool that is used to determine and estimate time derivative of arbitrary input signal and provide output signal in time domain. It is a dominating tool and used in all dimension of Science and Engineering. It has been used in many areas such as signal peak detection and slope recognition in mathematical processing. Differentiators have been used in several areas such as image processing, speech system and pulse generation [1]-[10]. They are integral part in analysis of signals in radar and sonar system [1], biomedical engineering application system [2] for computing and processing higher order derivative. Particularly, major importance has been observed in reconfigurable pulse shapers [4] and microwave waveform generation [5].

Several methods for differentiation operation are discussed, taking into consideration the bandwidth and frequency range. For achieve higher frequency range of operation, digital design techniques have been considered that include design of finite impulse response (FIR) and infinite impulse response (IIR) differentiators [7]-[12]. A close-form method for design digital differentiator using an eigen filter approach was proposed [13]. Al- Alaoui [7] used Simpson's rule to design a stable second order recursive differentiator. Tseng [8] discussed differentiator design by using Vandermonde method. Khan and Ohba [11] used central difference approximations for maximum linear differentiators. A second-order digital differentiator was obtaining by inverting the transfer function of integrator and stabilizing them [13]. Significant number of research is going on to use of photonic technologies for differentiator design at microwave frequency range. Here, the main concern is integration of optical device with microwave device which added technical complexity.

Taking into account the current interest micro-strip transmission line microwave differentiator will be surveyed here. A first-order differentiator for a limited bandwidth has been proposed and simulation by Tsai and Jeng [14]. An approximated solution in the Z-domain was studied by Hsue *et al.* in [7] to implement first- and second- order differentiator.

In this paper, novel differentiator design technique is discuss by means of comparison of transfer function define and develop in the Digital signal Processing (DSP) study of differentiator with the transfer function develop by cascade connection of transmission-line configurations. Z-parameter is used to define the scattering characteristics of equal electrical length transmission line [15], [16]. As a result, various combination of equal electrical length transmission-line will produce transfer function in Z-domain very much similar to the transfer function obtain from DSP study in Z- domain. Then higher order differentiators are design using micro-strip line based on the micro-strip line parameter values obtain by comparisons of transfer functions.

It is therefore, fabricate differentiator having operating frequency lies between 1-10 GHz. The close agreement between the simulation results, obtain from theoretical values and simulated on Advance Design Value (ADS) and the experimental results further validates the design. It is pertinent to point out that will be some degree of variation after fabrication due to limitation of fabrication process and material availability.

II. Synthesis Method Of Discrete-Time Differentiators

Z- Parameter is very useful tool for discrete-time analysis of a filter. For instant, Laplace transform representation of time derivative of any signal is given by complex frequency variable, as $s = \sigma + j\omega$; while neglecting loss term, we have $s = j\omega$. This is simply implies that transfer function of a differentiator is directly proportional to the signal angular frequency in Laplace domain and magnitude is linearly increases with the signal angular frequency. Before proceed to future analysis, system transfer function is transform from S-domain to Z- domain.

$$s = \frac{2(1 - z^{-1})}{T(1 + dz^{-1})} \quad (1)$$

For that we learn various transformation tools but use bilinear transformation for $d = 1$ in (1). As bilinear transformation shows linearity only for lower side of frequency range we made small modification. Here d is real constant that choose to be 0.165 [33] that ensure very good linearity for desired frequency range, which is use many place in converting analog prototype to discrete-time prototype [10]. Where T is normalized constant and z^{-1} represent a unit of time delay and given by $z = e^{j\Omega}$, It is required that the amplitude response of (1) should be less then unity for the entire frequency range. So $2/T$ is set equal to 0.417. So we can now define system transfer function of differentiators for first order, second order and higher order also. We will limit to our discussion only to second order and its designs. System function of a differentiator in discrete-time IIR format selected as,

$$\mathrm{G(z)} = \left[\frac{0.417(1 - \mathrm{z}^{-1})}{(1 + .1658dz^{-1})} \right] \quad (2)$$

By implementing the circuit using system function shown in (2), the differentiator gives close agreement with ideal possible design.

For a second-order differentiator, the system function $H(z)$ is obtained by squaring $G(z)$ and it is given by,

$$\mathrm{H(z)} = \left[\frac{0.417(1 - \mathrm{z}^{-1})}{(1 + .1658dz^{-1})} \right]^2 \quad (3)$$

After selecting the proper system transfer function of both orders, we will concentrate our discussion to Z-transform parameter for equal electrical length transmission line.

III. Z- Transform Parameter Of Micro-Strip Lines

For a filter design operating in microwave frequency range cannot be synthesize and analysis using low frequency parameters. We have scattering matrix and transfer matrix parameters that can be used for filter design at microwave frequency range. But here transfer matrix (T-matrix) is convenience to use in differentiator design because of it is design by means of cascade connection of equal electrical length transmission line. And by knowing the T-matrix of individual elements used in design, T-matrix of the implemented design will obtain by multiplication of T-matrix of each element and that will help us to obtain the transfer function of the implemented design as discuss below.

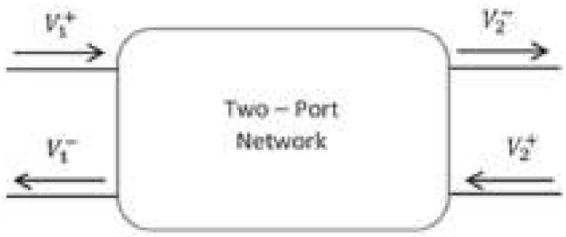

Fig. 1: Two-Port Network

The transfer function of a cascaded network can be found by multiplying the chain scattering matrices of the components composing the network. The chain scattering parameters T_{mn},m, n = 1,2 of a two-port network are defined by assuming the waves V_1^+ and V_2^-at port 1 in Fig. 1 are dependent variables, and the waves and at port 2 are independent variables. The T matrix (transfer matrix), which directly relates the waves on the input and on the output, is defined as;

$$\begin{bmatrix} V_1^+ \\ V_1^- \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} V_2^+ \\ V_2^- \end{bmatrix} \quad (4)$$

As the transmission matrix (T matrix) simply links the input and outgoing waves in a way different from the S matrix, one may convert the matrix elements mutually. The chain scattering matrix can be found from the scattering matrix in the following way,

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{S_{21}} & -\frac{S_{22}}{S_{21}} \\ \frac{S_{11}}{S_{21}} & S_{21} - \frac{S_{11}S_{22}}{S_{21}} \end{bmatrix} \quad (5)$$

Let the length of all stubs and transmission-line sections be $l = \lambda_o/4$, where λo is the wavelength of the lines at the normalizing angular frequency ω_o. In other words, the electrical length of all components is 90° at the normalizing frequency. The T-matrix of a cascade network consist of series-shunt transmission-line [8], [9] is given by,

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}_{Network} = \prod_{i=1}^{N} \begin{bmatrix} T_{11}^i & T_{12}^i \\ T_{21}^i & T_{22}^i \end{bmatrix} \quad (6)$$

Where, N is the number of the components, and T_{11}^i,T_{12}^i ,T_{21}^i,$T_{22}^i$and are the matrix elements representing the i_{th} component. T Matrix in Z-domain for series and shunt transmission line are given in Table 1. Note that Z_o is the reference characteristics impedance, which is assume to be 50 Ω, until otherwise mentioned.

Table 1. Basic Transmission-Line Element's Chain Scattering-Parameter Matrices

Transmission line Configuration	T- parameter
Serial Transmission line	$\frac{1}{z^{-1/2}(1-\Gamma^2)}\begin{bmatrix}1-\Gamma^2 z^{-1} & -(\Gamma-\Gamma z^{-1}) \\ \Gamma-\Gamma z^{-1} & -\Gamma^2+z^{-1}\end{bmatrix}$ where, $\Gamma_m=\frac{Z_2-Z_1}{Z_2+Z_1}$
Shunt-Short stub	$\frac{1}{1-z^{-1}}\begin{bmatrix}(1+a)-(1-a)z^{-1} & a+az^{-1}) \\ -a-az^{-1} & (1-a)-(1+a)z^{-1}\end{bmatrix}$ where, $a=\frac{Z_0}{2Z_1}$
Shunt-Open stub	$\frac{1}{1+z^{-1}}\begin{bmatrix}(1+a)+(1-a)z^{-1} & a-az^{-1}) \\ -a+az^{-1} & (1-a)+(1+a)z^{-1}\end{bmatrix}$ where, $a=\frac{Z_0}{2Z_1}$

IV. Implementation Of Second Order Differentiator

A system function of overall transmission line network terminated with matched load is represented $S_{21}(z)$ is given by $1/T_{11}(z)$. The chain-scattering parameter matrix element $T_{11}(z)$ for overall network is given in (7). From Table 1, for a micro-strip line configuration consist of M series section, K shunt open section and L shunt short section, generalized transfer function of configuration is shown in (9). Where, α_i are real and determine by the characteristics impedance of all transmission line elements and Γ_m is reflection coefficient for transmission line section define in Table 1.

Generalized transfer function of configuration is shown below:

$$T_{11}(z)=\frac{\sum_{i=0}^{N}\alpha_i z^{-i}}{\prod_{k=1}^{K}(1+z^{-1})\prod_{l=1}^{L}(1-z^{-1})\prod_{m=1}^{M}(z^{-1}(1-\Gamma_m^2))} \quad (7)$$

$$S_{21}(z)=T(z)=\left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}=\frac{1}{T_{11}(z)} \quad (8)$$

$$\frac{1}{T_{11}(z)}=\frac{\prod_{k=1}^{K}(1+z^{-1})\prod_{l=1}^{L}(1-z^{-1})\prod_{m=1}^{M}(z^{-1/2}(1-\Gamma_m^2))}{\sum_{i=0}^{N}A_i z^{-i}} \quad (9)$$

Where, $A_i=\frac{\alpha_i}{\prod_{m=1}^{M}(z^{-1/2}(1-\Gamma_m^2))}$ are functions of the characteristic impedances of both stubs and transmission-line sections. By taking into consideration of transfer function in (8), it is sufficient to assume shunt short transmission line for differentiator design and transfer function after neglecting the delay term, the transfer function will define as:

$$S_{21}=\frac{(1-z^{-1})^L\prod_{m=1}^{M}(1-\Gamma_m^2)}{\prod_{l=1}^{L}\left[(1+a)-(1-a)z^{-1}\right]\left[\prod_{m=1}^{M}1-\Gamma_m^2 z^{-1}\right]} \quad (10)$$

For designing a differentiator that have the operating frequencies up to 10GHz using micro-strip configuration, a network consist of a short stub and a serial transmission lines is used. Again neglecting the delay term define in (10), a general combination of micro-strip line element transfer function (10) will be used to equate with the second order digital differentiator system transfer function (3).

$$\left[\frac{0.417\left(1-z^{-1}\right)}{\left(1+.1658z^{-1}\right)}\right]^2=\frac{(1-z^{-1})^L\prod_{m=1}^{M}(1-\Gamma_m^2)}{\prod_{l=1}^{L}\left[(1+a)-(1-a)z^{-1}\right]\left[\prod_{m=1}^{M}1-\Gamma_m^2 z^{-1}\right]} \quad (11)$$

Now the remaining task is to determine the best possible value for all parameters for various combinations of L and M which will Advance Design System (ADS) Tool is now use for Simulation of differentiator based on the parameter obtained by the comparison of both transfer function. Physical dimension of second order differentiator listed in Table 2 is obtain form the Line Cal of ADS with same number of series (L=3) and shunt transmission line (M=3) used.

The RT/duroid® 5880 is used as dielectric substrate having a thickness of 15 mil (0.381mm) and relative dielectric constant of ε = 2.2. And based on the above dimension the schematic design in Fig. 2 and layout design in Fig. 3 for microwave differentiator is obtained using schematic and layout window of Agilent ADS tool respectively. Whereas