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Quantum teleportation using entangled 3-qubit states and the 'magic bases'

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ABSTRACT

We study quantum teleportation of single qubit information state using 3-qubit general entangled states. We propose a set of 8 GHZ-like states which gives (i) standard quantum teleportation (SQT) involving two parties and 3-qubit Bell state measurement (BSM) and (ii) controlled quantum teleportation (CQT) involving three parties, 2-qubit BSM and an independent measurement on one qubit. Both are obtained with perfect success and fidelity and with no restriction on destinations (receiver) of any of the three entangled qubits. For SQT, for each designated one qubit which is one of a pair going to Alice, we obtain a magic basis containing eight basis states. The eight basis states can be put in two groups of four, such that states of one group are identical with the corresponding GHZ-like states and states of the other differ from the corresponding GHZ-like states by the same phase factor. These basis states can be put in two different groups of four-states each, such that if any entangled state is a superposition of these with coefficients of each group having the same phase, perfect SQT results. Also, for perfect CQT, with each set of given destinations of entangled qubits, we find a different magic basis. If no restriction on destinations of any entangled qubit exists, three magic semi-bases, each with four basis states, are obtained, which lead to perfect SQT. For perfect CQT, with no restriction on entangled qubits, we find four magic quarter-bases, each having two basis states. This gives perfect SQT also. We also obtain expressions for co-concurrences and conditional concurrences.

1. Introduction

Quantum entanglement [1–7] is a main resource of quantum information theory. It is widely used in quantum information processing tasks like quantum teleportation [8], quantum computation [9], quantum superdense coding [10], quantum cryptography [11] and secure quantum information exchange [12]. A pure state of two or more quantum systems is said to be entangled [3], if it cannot be written as product of the quantum states of constituent systems. A mixed state is called entangled [3], if it cannot be written as a mixture of factorizable pure states, i.e., it cannot be written in the form $\rho=\sum_j p_j\rho_j^{A_1}\otimes\rho_j^{A_2}\otimes.....\otimes\rho_j^{A_n}$ with $p_j>0$ and $\sum_j p_j=1$.

For a bipartite system, Hill and Wootters [2,3] gave a measure of entanglement called Concurrence, defined as,

$$C(\rho)=\ \max\{0,\lambda_1-\lambda_2-\lambda_3-\lambda_4\},$$

where ρ is the density operator of the system, λ_1, λ_2, λ_3 and λ_4 are non negative square roots of eigenvalues of the matrix $\rho\tilde{\rho}$ in decreasing order, $\tilde{\rho}$ is density operator for spin flipped state, defined as, $\tilde{\rho}=(\sigma_y\otimes\sigma_y)\rho^*(\sigma_y\otimes\sigma_y)$. $C(\rho)$ can take values from 0 to 1. $C(\rho)=1$ implies maximal entanglement, while $C(\rho)=0$ implies no entanglement. For three qubit pure states, Coffman et al. [4] proved that $C_{12}^2+C_{13}^2\le C_{1(23)}^2$ (called CKW inequality), where C_{12}, C_{13} and $C_{1(23)}$ are concurrences between particles 1 and 2, 1 and 3 and 1 and pair 23. From this inequality it is clear that for a three qubit pure state qubit 1 has a certain amount of entanglement with the pair 23 that restricts the entanglement of qubit 1 with qubits 2 and 3 considering them individually. Further, the authors defined a quantity called three tangle [4,5] $\tau\equiv C_{1(23)}^2-C_{12}^2-C_{13}^2$, which comes naturally from CKW inequality and is invariant under permutation of qubits. Three tangle represents the residual entanglement of the three qubit pure state and is a useful measure of entanglement. Soojoon Lee et al. [6] defined other three quantities similar to three tangle, called the partial tangles for a three qubit pure state, and found that partial tangle is closely related to teleportation fidelity of single qubit information state using three qubit pure entangled state.

Quantum teleportation (QT), first due to Bennett el al [8], is a phenomenon in which unknown quantum information state is destroyed at the sender's end and a replica is created at receiver's end using long range EPR correlations [1], also called as quantum entanglement. Bennett et al. [7] further proposed a set of four bipartite mutually orthogonal maximally entangled states, whose arbitrary linear superposition leads to perfect teleportation whenever modulus of sum of squares of coefficients of the states is unity. Since sum of squares of moduli of the coefficients is also unity because of normalization, this demands same phase of all coefficients. Thus,

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any linear superposition of the states in this basis with coefficients having a global phase gives an entangled state which leads to perfect teleportation. Such a basis was called "magic basis" by Hill and Wootters [2]. A number of studies on quantum teleportation [13–28] have been published since then but existence of magic basis has not been reported in any case other than that of Bennett et al.'s [7] entangled two qubits.

Experimental demonstration of quantum teleportation has been done with single photon state using a pair of entangled photons [13,14] and an atomic qubit state using two entangled qubits [15,16]. Rigolin [17] proposed faithful teleportation of two qubit information using four qubit entangled states. Author [17] defined 16 orthogonal generalized Bell states of four qubits and used them for the teleportation of two qubit information. Rigolin's generalized Bell states are, however, factorizable and, therefore, qubits going to Alice and Bob are fixed, and not permutable [18]. In all these studies, qubits correspond to quantum states of two level systems. An alternate scheme involving coherent states has also been used by several authors [19].

H. Prakash et al. [20] showed rigorously that QT of N qubit state requires an entangled state of at least 2N qubits. This implies that QT of single qubit information state requires an entangled state of at least 2 qubits and if we use three or more qubit entangled state, it will be a luxury. QT of single qubit information state using three qubit entangled state can be perceived with two possible schemes. In the first, Alice (sender) gets two qubits of the shared entangled state, while Bob (receiver) gets one qubit. Alice performs Bell state measurement (BSM) involving 3-qubits on her 3 qubits and conveys result to Bob by a classical channel. On the basis of this Bob performs the required unitary transformation on his qubit to get exact replica of information state. This process is called standard quantum teleportation (SQT). In the second scheme, Alice, Bob and an extra participant, Charlie, share the entangled state. Alice performs standard two qubit BSM in the Bell basis on her two qubits and Charlie measures his particle in the orthogonal basis of single qubit. Both Alice and Charlie convey their measurement results to Bob on the basis of which Bob performs a unitary transformation on his particle to generate an exact replica of the information state. Involvement of an extra person Charlie increases the security and the process is called controlled quantum teleportation (CQT). In this scheme Charlie has a control on teleportation process and works as a supervisor. Without Charlie's involvement, Bob can never generate the replica of the original information. Here, if, anyone is able to thieve the information and particles of Alice and Bob successfully, even then he can never generate the information because Charlie's result is also needed for this. CQT may also be useful in future quantum computers.

Teleportation of single qubit information using three qubit entangled states has been considered by several authors [21–27]. A. Karlsson et al. [21] and Hillery et al. [22] showed that quantum teleportation of single qubit information using an entangled GHZ state is possible, when Alice performs two particle BSM in the Bell basis and another observer performs one particle independent measurement in basis $([|0\rangle+|1\rangle]/\sqrt{2},[|0\rangle-|1\rangle]/\sqrt{2})$. In Refs. [23–25], authors presented schemes for quantum teleportation of single qubit information state using entangled W state and the maximum value of total success probability was found to be 2/3 for these schemes. Agrawal and Pati [26] proposed use of a W class state as entangled state and a set of four W class states for BSM in teleportation of single qubit information with perfect success and fidelity. These states are not symmetric under permutation of qubits and teleportation fails if qubit going to Bob is other than the one originally proposed.

Wang et al. [27] has very recently presented in this journal two schemes of teleportation of single qubit information state using the same GHZ state for both schemes but different BSMs. In the first scheme, the authors used GHZ states for BSM and, in the second, they proposed a set of 8 other states of three qubits for BSM. These other states are, however, separable between particles 13 and 2. Thus, these states have only two particle entanglement, i.e., entanglement between 1 and 3 and no three particle entanglement exists. This teleportation scheme involves BSM performed by Alice consisting of two particles BSM in the Bell basis and also one particle independent measurement in the basis $([|0\rangle+|1\rangle]/\sqrt{2},[|0\rangle-|1\rangle]/\sqrt{2})$, which is exactly similar to teleportation scheme presented by Hillery et al. [22], which involved three parties. Entangled GHZ states and GHZ class states have also been used for QT of single qubit information involving two qubit entangled state [28].

In the present paper, we make detailed investigation of SQT and CQT using a proposed set of 8 GHZ-like states of three qubits belonging to GHZ class. These GHZ-like states are convertible into GHZ states using LOCC [29]. We find that both SQT and CQT of single qubit information using anyone of the GHZ-like states as an entangled state give perfect success and fidelity, without any restriction whatsoever on particles going to various parties.

We show that, for perfect SQT, for each one designated qubit which is one of a pair going to Alice, a magic basis with eight basis states exists. Since there can be three cases of one designated qubit which is one of a pair going to Alice, we get three sets of magic bases for three qubits. For perfect CQT, similarly, we obtain three magic bases, one each for a designated qubit going to Charlie, independent of how the remaining two entangled qubits are distributed between Alice and Bob.

However, if the destinations of qubits are not fixed, for SQT, no magic basis exists, but three sets of four states exist, which we call as magic semi-bases. These sets of four states are said to form magic semi-bases as any set of four states is incomplete in the 8-dimensional Hilbert space of 3 qubits. Superposition of the four states of any magic semi-basis with coefficients having a global phase gives SQT with perfect success and fidelity without any restriction on qubits and parties. For perfect CQT, if the destinations of qubits are not fixed, we find four sets of two basis states, which we call magic quarter-basis. These also give perfect SQT. It may be noted that after introduction of magic basis by Bennett et al. [7] for the simplest case of two qubits, this seems first reported case on existence of magic bases of three qubits for SQT and CQT.

2. GHZ-like states for three qubits and standard and controlled QT

For three qubits, we find that one simple way of writing the 8 GHZ-like states is

$$|G\rangle^{(1,2)} = \frac{1}{2}[|000\rangle + |011\rangle \pm |101\rangle \pm |110\rangle], \tag{1}$$

$$|G\rangle^{(3,4)} = \frac{1}{2}[|000\rangle - |011\rangle \pm |101\rangle \mp |110\rangle], \tag{2}$$

$$|G\rangle^{(5,6)} = \frac{1}{2}[|111\rangle + |100\rangle \pm |010\rangle \pm |001\rangle], \tag{3}$$

$$|G\rangle^{(7,8)} = \frac{1}{2}[|111\rangle - |100\rangle \pm |010\rangle \mp |001\rangle]. \tag{4}$$

These GHZ-like states form an orthonormal and complete basis

$${}^{(i)}_{123}\langle G|G\rangle^{(j)}_{123} = \delta_{ij} \text{ and } \sum_{i=1}^{8} |G\rangle^{(i)}_{123}\,{}^{(i)}_{123}\langle G| = I. \tag{5}$$

On exchange of first and second qubits, we get, $|G\rangle^{(r)}_{123} = |G\rangle^{(s)}_{213}$ with $(r,s) = (1,1)$, (2,3), (3,2), (4,4), (5,5), (6,7), (7,6), and (8,8) and on exchange of second and third, we get $|G\rangle^{(r)}_{123} = |G\rangle^{(s)}_{132}$ with $(r,s) = (1,1)$, (2,2), (3,4), (4,3), (5,5), (6,6), (7,8), (8,7).

It should be noted that the above GHZ-like states are convertible into GHZ states using Hadamard operations on each qubit. These