J. Phys. B: At. Mol. Opt. Phys. **44** (2011) 115504 (8pp) doi:10.1088/0953-4075/44/11/115504

Two-way quantum communication: 'secure quantum information exchange'

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Received 22 March 2011, in final form 16 April 2011
Published 19 May 2011
Online at stacks.iop.org/JPhysB/44/115504

Abstract
In this paper, we present a new idea of two-way quantum communication called 'secure quantum information exchange' (SQIE). If there are two arbitrary unknown quantum states $|\xi\rangle_A^I$ and $|\eta\rangle_B^I$, initially with Alice and Bob, respectively, then SQIE protocol leads to the simultaneous exchange of these states between Alice and Bob with the aid of the special kind of six-qubit entangled (SSE) state and classical assistance of the third party, Charlie. The term 'secure' signifies the fact that SQIE protocol either faithfully exchanges the unknown quantum states proceeding in a prescribed way or, in case of any irregularity, the process generates no results. For experimental realization of the SQIE protocol, we have suggested an efficient scheme for generating SSE states using the interaction between highly detuned Λ-type three-level atoms and the optical coherent field. By theoretical calculations, we found that SSE states of almost unit fidelity with perfect success rates for appreciable mean photon numbers ($F_{\text{av}} \geqslant 0.999$ for $|\alpha|^2 \geqslant 1.5$) can be generated by our scheme. Further, we have discussed possible experimental imperfections, such as atomic-radiative time, cavity damping time, atom–cavity interaction time, and the efficiency of discrimination between the coherent field and the vacuum state shows that our SQIE protocol is within the reach of technology presently available.

1. Introduction

Quantum entanglement [1] plays an important role in the quantum information processing tasks such as quantum teleportation (QT) [2], quantum cryptography [3], quantum super-dense coding [4], quantum remote state preparation [5] and many more. Since the no-cloning theorem [6] forbids the creation of identical copies of an arbitrary unknown quantum state, to deal with unknown quantum states (inaccessible information, by measurement we cannot extract complete information encoded in a quantum state) for using them in different information processing tasks, sometimes it is required to map the quantum state from one particle to another particle. Mapping of the quantum state, $|\xi\rangle_1 = [a_0|0\rangle + a_1|1\rangle]_1$, from one quantum particle 1 to another particle 2 initially in the state $|0\rangle_2$ can be performed by applying the C-Not gate [7] with control as the information particle 1 state,

$$(a_0|0\rangle + a_1|1\rangle)_1 \otimes |0\rangle_2 \xrightarrow{\text{C-Not}} a_0|00\rangle_{12} + a_1|11\rangle_{12},$$

and the subsequent measurement of particle 1 in the diagonal basis $(1/\sqrt{2})[|0\rangle \pm |1\rangle]$. Also it is possible to map the information state from an atom to radiation or from radiation to the atom using atom–field cavity interaction [8]. However, if we consider two quantum processors working far apart, then to set a link between these two processors, we need to have information mapping from the first processor particle to the second processor particle across space. For this purpose, the QT scheme, first proposed by Bennett *et al* [2], is surprisingly useful as it allows the mapping of quantum information encoded in an unknown quantum state of a particle to another particle across space with the aid of quantum entanglement [1], without physically sending any particle. It is well known that QT can be used for the construction of quantum gates [9] using single photons as qubits, and hence is expected to be an effective tool for the realization of quantum computers and also proved to be an effective technique for quantum cryptography [10].

For these reasons, large numbers of schemes have been proposed for realizing the QT of an unknown information state encoded in atomic, polarized-photonic, ionic and superposed coherent states [11–14]. Some authors have experimentally demonstrated the QT of single-qubit information encoded in an atom [12], a polarized photon [13], a nuclear spin [14], etc. Pati *et al* [15] studied the problem of the QT of a single-qubit state using non-maximally entangled states and proposed probabilistic teleportation. All of these relevant works involve bipartite Bell-state measurement (BSM) and the transfer of 2-cbit information from the sender (Alice) to the receiver (Bob). Wang and Pati *et al* [16] teleported the single-qubit information state via the tripartite GHZ and W states, respectively, using tripartite joint measurement. However, many authors [17] by introducing a third party and using the GHZ or W state proposed secure quantum communication known as controlled QT.

Further, many authors in [18, 20] teleported the two-qubit state of the form

$$|\xi\rangle = [a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle], \quad (1)$$

using a 4-particle entangled state and a 4-cbit classical channel. Rigolin [19] used a 4-particle entangled state of the form

$$|g\rangle_{A_1,A_2,B_1,B_2} = \frac{1}{2}\left[\sum_{\alpha_1,\alpha_2=0,1} |\alpha_1\alpha_2\rangle \otimes \sigma^{i_1} \otimes \sigma^{i_2}|\alpha_1\alpha_2\rangle\right]_{A_1,A_2,B_1,B_2}, \quad (2)$$

where $\sigma^{i_1}, \sigma^{i_2} \in [I, \sigma_z, \sigma_x, \sigma_x\sigma_z]$ and then generalized this for N-qubit teleportation using the $2N$-particle entangled state. However, it has been shown by Deng [19] that the state in (2) is the product of two EPR pairs, not a genuine multipartite entangled state; hence, this protocol in principle is equivalent to the protocol previously proposed by Yang and Guo [20].

All of these relevant works enable the one-way QT of the single-qubit or multi-qubit unknown state. However, for practical realization of a secure quantum network consisting of many quantum processors working far apart, there may be some situations when the simultaneous QT of information states from Alice to Bob, and at the same time from Bob to Alice, is required. However, one can say that this is possible simply by switching on two independent QT setups both in opposite directions, one from Alice to Bob and the other from Bob to Alice. Since standard QT schemes [2] need classical information about the BSM results from the sender to the receiver to be sent, a situation may arise when Alice's state gets teleported to Bob, but Bob denies to send the classical information to Alice. Hence Bob's state does not teleport to Alice, which gives rise to an insecure or dishonest quantum network. However, two-way QT proposed by Vaidman [2], that involves two nonlocal measurements, is capable of exchanging quantum states between Alice and Bob. But since it requires the transmission of measurement results from Alice to Bob and from Bob to Alice, it gives rise to an insecure quantum network whenever Alice or Bob denies sending measurement results. Moreover, two nonlocal measurements will require two EPR pairs and four local measurements, while we shall see that our protocol using the six-particle entangled state defined later in section 2, give secure quantum information exchange (SQIE) and involves only two local BSM. Also one cannot use two controlled QT setups in the opposite direction to exchange quantum states securely via the GHZ or W state because this would also require the transmission of classical information from Alice to Bob and from Bob to Alice, which will again give rise to an insecure quantum network. We shall see that this problem will not arise in our scheme. To overcome such a problem, in the light of the above discussion, we need to have a scheme with the qualities that (a) it can simultaneously teleport Alice's information to Bob and Bob's information to Alice, (b) security that the scheme either exchanges the unknown quantum states or, in case of any irregularity, it generates no results, i.e., neither Alice or Bob receives any information, and (c) it must be experimentally realizable with presently available technology. We call such two-way quantum communication the 'secure quantum information exchange' protocol. In the present contribution, we will propose an efficient scheme to realize SQIE, which is expected to fulfil all these requirements.

We organize the paper as follows: in section 2, we define a set of special kinds of six-qubit entangled (SSE) states and show how to implement them for SQIE. In section 3, we present an experimental scheme for generating SSE states using the interaction between Λ-type three-level atoms and the optical coherent field. In section 4, we discuss the experimental aspects of our protocol, and finally we summarize our results with a conclusion in section 5.

2. Secure quantum information exchange

Let us consider that Alice wants to send the information state in mode A, $|\xi\rangle_A^I = [a_0\,|0\rangle + a_1\,|1\rangle]_A$, to Bob and Bob wants to send the information state in mode B, $|\eta\rangle_B^I = [b_0\,|0\rangle + b_1|1\rangle]_B$, to Alice, with the security that either both get their required information state or, in case of any discrepancy, both do not get the required information state. For this, we define a set of SSE states as

$$|\psi\rangle^E = \frac{1}{2}\left[\sum_{i=1}^{4} |B\rangle^{(i)} \otimes |B\rangle^{(i)} \otimes |\phi\rangle^{(i)}\right] \quad (3)$$

where $|B\rangle^{(1,2)} = \frac{1}{\sqrt{2}}[|00\rangle \pm |11\rangle]$, $|B\rangle^{(3,4)} = \frac{1}{\sqrt{2}}[|01\rangle \pm |10\rangle]$ are the standard bipartite Bell states and $|\phi\rangle^{(i)} \in \{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. If we put an additional condition that all four $|\phi\rangle^{(i)} (i = 1, 2, 3, 4)$ are different, the states defined by equation (3) give a set of 24 SSE states.

Using one of these states,

$$\begin{aligned}|\psi\rangle^E_{A_1,B_1,A_2,B_2,C_1,C_2} = \tfrac{1}{2}[&|B\rangle^{(1)} \otimes |B\rangle^{(1)} \otimes |00\rangle \\ &+ |B\rangle^{(2)} \otimes |B\rangle^{(2)} \otimes |11\rangle \\ &+ |B\rangle^{(3)} \otimes |B\rangle^{(3)} \otimes |01\rangle \\ &+ |B\rangle^{(4)} \otimes |B\rangle^{(4)} \otimes |10\rangle]_{A_1,B_1,A_2,B_2,C_1,C_2}, \end{aligned} \quad (4)$$

the initial state of the composite system can be written as

$$\begin{aligned}|\psi\rangle_{A,A_1,B_1,A_2,B_2,C_1,C_2,B} &= |\xi\rangle_A^I \otimes |\psi\rangle^E_{A_1,B_1,A_2,B_2,C_1,C_2} \otimes |\eta\rangle^I_B \\ &= \tfrac{1}{2}\big[|\xi\rangle_A^I \otimes |B\rangle^{(1)}_{A_1,B_1} \otimes |B\rangle^{(1)}_{A_2,B_2} \otimes |00\rangle_{C_1,C_2} \otimes |\eta\rangle^I_B \\ &+ |\xi\rangle_A^I \otimes |B\rangle^{(2)}_{A_1,B_1} \otimes |B\rangle^{(2)}_{A_2,B_2} \otimes |11\rangle_{C_1,C_2} \otimes |\eta\rangle^I_B \\ &+ |\xi\rangle_A^I \otimes |B\rangle^{(3)}_{A_1,B_1} \otimes |B\rangle^{(3)}_{A_2,B_2} \otimes |01\rangle_{C_1,C_2} \otimes |\eta\rangle^I_B \\ &+ |\xi\rangle_A^I \otimes |B\rangle^{(4)}_{A_1,B_1} \otimes |B\rangle^{(4)}_{A_2,B_2} \otimes |10\rangle_{C_1,C_2} \otimes |\eta\rangle^I_B\big], \end{aligned} \quad (5)$$

where the subscripts A_1, A_2 refer to entangled modes with Alice, B_1, B_2 refer to entangled modes with Bob and C_1, C_2

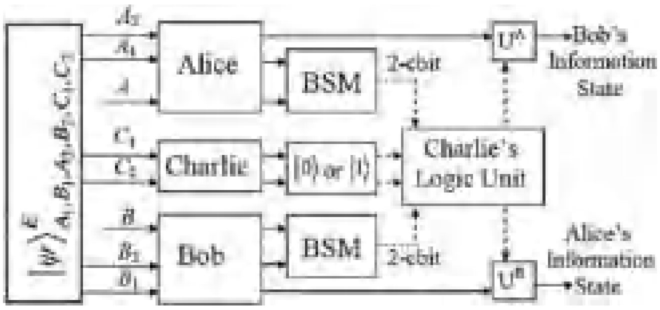

Figure 1. Scheme for SQIE. A, B are information modes with Alice and Bob, respectively. $|\psi\rangle^E_{A_1,B_1,A_2,B_2,C_1,C_2}$ is a SSE state. The entangled modes (A_1, A_2), (B_1, B_2) and (C_1, C_2) are with Alice, Bob and Charlie, respectively. BSM boxes refer to bipartite Bell-state measurement by Alice and Bob, while the box with state $|0\rangle$ or $|1\rangle$ refers to the measurement in the basis $\{|0\rangle, |1\rangle\}$. The logical unit processes the measurement results of Alice, Bob and Charlie for deciding the 2-cbit results to be conveyed to Alice and Bob. U^A and U^B refer to the unitary operations to be performed by Alice and Bob, respectively, to complete faithful SQIE.

refer to entangled modes with Charlie. The superscripts E and I refer to the entangled state and information states, respectively.

Our complete scheme for SQIE is shown in figure 1. Writing states in modes A, A_1 and B_2, B in the standard bipartite Bell basis $|B\rangle^{(j)}_{A,A_1}$ and $|B\rangle^{(k)}_{B_2,B}$, respectively, equation (5) further simplifies to

$$|\psi\rangle_{A,A_1,B_1,A_2,B_2,C_1,C_2,B} = \frac{1}{4}\left[\sum_{i,j,k=1}^{4}\left(|B\rangle^{(j)}_{A,A_1} \otimes U^{\dagger}_{ij}|\xi\rangle^{I}_{B_1}\right.\right.$$
$$\left.\left.\otimes U^{\dagger}_{ik}|\eta\rangle^{I}_{A_2} \otimes |B\rangle^{(k)}_{B_2,B} \otimes |\varphi\rangle^{(i)}_{C_1,C_2}\right)\right]. \quad (6)$$

where $U_{1j} = U_{1k} = (I, \sigma_z, \sigma_x, \sigma_z\sigma_x)$, $U_{2j} = U_{2k} = (\sigma_z, I, \sigma_z\sigma_x, \sigma_x)$, $U_{3j} = U_{3k} = (\sigma_x, -\sigma_z\sigma_x, I, -\sigma_z)$ and $U_{4j} = U_{4k} = (-\sigma_z\sigma_x, \sigma_x, -\sigma_z, I)$.

Now both Alice and Bob perform BSM on their qubits A, A_1 and B_2, B, respectively, while Charlie measures his qubits C_1, C_2 in the basis $\{|0\rangle, |1\rangle\}$. Both convey their BSM results to Charlie through 2-bit classical channels. Charlie on the basis of BSM results obtained by Alice and Bob and his measurement results decides the 2-bit classical information to be conveyed to Alice and Bob. On the basis of this classical information, conveyed by Charlie, Alice and Bob perform the required unitary transformation on their particles in order to generate exact replicas of corresponding quantum information states. In table 1, we tabulate all measurement results in terms of 2-cbit obtained by Alice, Bob, Charlie and 2-cbit that Charlie conveys to Alice and Bob to complete the process of quantum information exchange.

From equation (6) and table 1, it is clear that the unitary transformations required by Alice and Bob to exchange information states faithfully are completely dependent on Charlie's measurement result (classical information to be conveyed by Charlie to Alice and Bob, for each combination of Alice and Bob's BSM result, is different for each of Charlie's measurement results); therefore, Alice and Bob cannot exchange their information states without the assistance of Charlie. Also if either Alice or Bob, for some reason, denies to convey the BSM result, then Charlie disregards the complete protocol without sending any information to Alice and Bob. That is why we call this the 'secure quantum information exchange' protocol.

Table 1. Classical information conveyed by Charlie to 'Alice and Bob' depending upon his own measurement result and results conveyed by 'Alice and Bob'. Alice and Bob perform unitary operations $I, \sigma_z, \sigma_x, \sigma_x\sigma_z$ depending on the classical information 00, 01, 10, 11, respectively, conveyed by Charlie.

2-cbit information about BSM conveyed by Alice and Bob to Charlie		Charlie's decision about the 2-cbit information to be conveyed to Alice and Bob for completing the QIE protocol							
		Charlie's measurement result							
		$\|00\rangle$		$\|11\rangle$		$\|01\rangle$		$\|10\rangle$	
Alice	Bob	Alice	Bob	Alice	Bob	Alice	Bob	Alice	Bob
00	00	00	00	01	01	10	10	11	11
00	01	01	00	00	01	11	10	10	11
00	10	10	00	11	01	00	10	01	11
00	11	11	00	10	01	01	10	00	11
01	00	00	01	01	00	10	11	11	10
01	01	01	01	00	00	11	11	10	10
01	10	10	01	11	00	00	11	01	10
01	11	11	01	10	00	01	11	00	10
10	00	00	10	01	11	10	00	11	01
10	01	01	10	00	11	11	00	10	01
10	10	10	10	11	11	00	00	01	01
10	11	11	10	10	11	01	00	00	01
11	00	00	11	01	10	10	01	11	00
11	01	01	11	00	10	11	01	10	00
11	10	10	11	11	10	00	01	01	00
11	11	11	11	10	10	01	01	00	00

3. Generation of the SSE state

The next natural question is to generate SSE states (equation (3)) that are used as a quantum channel in our SQIE protocol. In this section, we present an efficient scheme for the generation of the SSE state (3). We consider the interaction of a Λ-type three-level atom with the optical coherent field. The level configuration of the atom is shown in figure 2, where $|0\rangle$ and $|1\rangle$ are two degenerate ground levels and $|2\rangle$ is the excited level. The frequency of the optical coherent field (ω_c) is largely detuned from the atomic transition frequency ω_0, i.e. $\Delta = \omega_0 - \omega_c$ is large. In a large detuning limit, the excited state $|2\rangle$ can be adiabatically eliminated during the interaction and the effective Hamiltonian can be expressed as [21]

$$H = -\lambda a^{\dagger}a[|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| + |1\rangle\langle 1|], \quad (7)$$

where $\lambda = g^2/\Delta$. Here we have assumed that coupling strengths between the cavity mode and atomic transition ($|0\rangle \to |2\rangle$; $|1\rangle \to |2\rangle$) are equal and are described by the coupling constant g, governed by the above Hamiltonian; the state of the system initially in the state $|0, \alpha\rangle$ or $|1, \alpha\rangle$ evolves in the following way:

$$|0,\alpha\rangle \xrightarrow{U(t)} \tfrac{1}{2}[|0\rangle(|\alpha\rangle + |\alpha\,\mathrm{e}^{2\mathrm{i}\lambda t}\rangle) - |1\rangle(|\alpha\rangle - |\alpha\,\mathrm{e}^{2\mathrm{i}\lambda t}\rangle)],$$
$$|1,\alpha\rangle \xrightarrow{U(t)} \tfrac{1}{2}[|1\rangle(|\alpha\rangle + |\alpha\,\mathrm{e}^{2\mathrm{i}\lambda t}\rangle) - |0\rangle(|\alpha\rangle - |\alpha\,\mathrm{e}^{2\mathrm{i}\lambda t}\rangle)]. \quad (8)$$

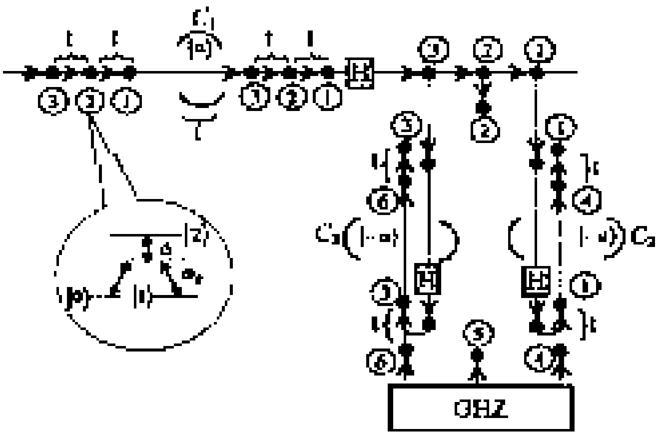

Figure 2. Scheme for the generation of SSE states (see equations (3) and (4)). The inset shows the level configuration of the three-level atom, where $|0\rangle$ and $|1\rangle$ are two degenerate ground levels and $|2\rangle$ is the excited level. C_1, C_2 and C_3 refer to the cavities initially prepared in the optical coherent fields $|\alpha\rangle_{C_1}$, $|-\alpha\rangle_{C_2}$ and $|-\alpha\rangle_{C_3}$, respectively. Encircled numbers 1, 2, ..., 6 denote atom 1, atom 2, ..., atom 6, respectively. Atoms 1, 2 and 3 are initially in the ground state $|000\rangle_{A_1,A_2,A_3}$, while atoms 4, 5, 6 are initially prepared in the GHZ state ($\sim [|000\rangle + |111\rangle]_{A_4,A_5,A_6}$). $t = \pi/2\lambda$ is the interaction time of the atom with cavity field. H refers to the Hadamard operation. For more details see the text.

If we select the atomic velocity such that the interaction time satisfies $t = \pi/2\lambda$, then

$$\begin{aligned} |0,\alpha\rangle &\xrightarrow{\lambda t=\pi/2} \tfrac{1}{2}[|0,+\rangle - |1,-\rangle]; \\ |1,\alpha\rangle &\xrightarrow{\lambda t=\pi/2} \tfrac{1}{2}[|1,+\rangle - |0,-\rangle], \end{aligned} \tag{9}$$

where $|\pm\rangle = [|\alpha\rangle \pm |-\alpha\rangle]$. Similarly if the system is initially in the state $|0,-\alpha\rangle$ or $|1,-\alpha\rangle$ and satisfies the interaction time $t = \pi/2\lambda$, the state of the system evolves to

$$\begin{aligned} |0,-\alpha\rangle &\xrightarrow{\lambda t=\pi/2} \tfrac{1}{2}[|0,+\rangle + |1,-\rangle]; \\ |1,-\alpha\rangle &\xrightarrow{\lambda t=\pi/2} \tfrac{1}{2}[|1,+\rangle + |0,-\rangle]. \end{aligned} \tag{10}$$

From relations (9) and (10), we obtain

$$\begin{aligned} &|0,+\rangle \to |0,+\rangle, |1,+\rangle \to |1,+\rangle, \\ &|0,-\rangle \to -|1,-\rangle, |1,-\rangle \to -|0,-\rangle, \end{aligned} \tag{11}$$

$$|A_+, \pm\alpha\rangle \to |A_+, \mp\alpha\rangle, |A_-, \pm\alpha\rangle \to |A_-, \pm\alpha\rangle, \tag{12}$$

where $|A_\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$.

Our detailed scheme for generating the SSE state is shown in figure 2. Let us prepare cavities C_1, C_2 and C_3 in the optical coherent field states $|\alpha\rangle_{C_1}$, $|-\alpha\rangle_{C_2}$ and $|-\alpha\rangle_{C_3}$, respectively, and three atoms in modes A_1, A_2 and A_3 in the states $|0\rangle_{A_1}$, $|0\rangle_{A_2}$ and $|0\rangle_{A_3}$, respectively. The initial state of the atom–cavity system is

$$|\psi(0)\rangle_{A_1,A_2,A_3,C_1,C_2,C_3} = |000\rangle_{A_1,A_2,A_3} \otimes |\alpha, -\alpha, -\alpha\rangle_{C_1,C_2,C_3}. \tag{13}$$

After the first atom A_1 has interacted with the coherent field in cavity C_1 for the interaction time $t = \pi/2\lambda$, atoms A_2 and A_3 are sent one by one through cavity C_1. If the interaction time still satisfies $t = \pi/2\lambda$ for both atoms A_2 and A_3, the state of the system evolves according to the evolution in equations (9)–(11), giving

$$\begin{aligned} &|\psi(3\pi/2\lambda)\rangle_{A_1,A_2,A_3,C_1,C_2,C_3} \\ &\quad = \tfrac{1}{2}[|000+\rangle - |111-\rangle]_{A_1,A_2,A_3,C_1} \otimes |-\alpha, -\alpha\rangle_{C_2,C_3}. \end{aligned} \tag{14}$$

Now we will complete the rotational operation (Hadamard operation) $R = \frac{1}{\sqrt{2}}\left(\begin{smallmatrix}1 & 1\\ 1 & -1\end{smallmatrix}\right)$, with $|0\rangle = \left(\begin{smallmatrix}1\\0\end{smallmatrix}\right)$ and $|1\rangle = \left(\begin{smallmatrix}0\\1\end{smallmatrix}\right)$, on each atom A_1, A_2 and A_3 to obtain

$$\begin{aligned} |\psi\rangle_{A_1,A_2,A_3,C_1,C_2,C_3} &= \tfrac{1}{2}[|A_+A_+A_+,+\rangle \\ &\quad - |A_-A_-A_-,-\rangle]_{A_1,A_2,A_3,C_1} \otimes |-\alpha,-\alpha\rangle_{C_2,C_3}. \end{aligned} \tag{15}$$

On sending atoms A_1 and A_3 through cavities C_2 and C_3, respectively, for the interaction time $t = \pi/2\lambda$, the state of the system evolves according to evolutions in equation (12), giving

$$\begin{aligned} |\psi\rangle_{A_1,A_2,A_3,C_1,C_2,C_3} &= \tfrac{1}{2}[|A_+A_+A_+,+,\alpha,\alpha\rangle \\ &\quad - |A_-A_-A_-,-,-\alpha,-\alpha\rangle]_{A_1,A_2,A_3,C_1,C_2,C_3}. \end{aligned} \tag{16}$$

Now again we will complete the rotational operation R on each atom A_1, A_2 and A_3, which gives

$$\begin{aligned} |\psi\rangle_{A_1,A_2,A_3,C_1,C_2,C_3} &= \tfrac{1}{2}[|000,+,\alpha,\alpha\rangle \\ &\quad - |111,-,-\alpha,-\alpha\rangle]_{A_1,A_2,A_3,C_1,C_2,C_3}. \end{aligned} \tag{17}$$

From equation (14), we see that the GHZ state $\left(\frac{1}{\sqrt{2}}[|000\rangle \pm |111\rangle]\right)$ can be generated simply by performing measurement on the cavity field in the basis $|\alpha\rangle$ and $|-\alpha\rangle$. However, the coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ are not orthogonal, $\langle\alpha \mid -\alpha\rangle = \mathrm{e}^{-2|\alpha|^2}$, but become orthogonal for large $|\alpha|^2$, i.e. for a large mean photon number. To distinguish $|\alpha\rangle$ and $|-\alpha\rangle$, we inject $|\alpha\rangle$ into the cavity, i.e. we make use of the displacement operator $D(\beta)\,|\alpha\rangle = |\alpha + \beta\rangle$. This gives $|\alpha\rangle \xrightarrow{(D\alpha)} |2\alpha\rangle$ and $|-\alpha\rangle \xrightarrow{(D\alpha)} |v\rangle$, where v corresponds to vacuum. Thus, for large $|\alpha|^2$, the state $|2\alpha\rangle$ has a very small probability of having zero photon and hence gives the non-zero number of photons on photon counting, while the state $|v\rangle$ gives zero count. In this way, we can generate the GHZ state, $\frac{1}{\sqrt{2}}[|000\rangle + |111\rangle]$, with another setup. We will use this GHZ state as an ancillary state in modes A_4, A_5, A_6. Now a complete state of the system is the product of the state in equation (17) and the GHZ state,

$$\begin{aligned} &|\psi\rangle_{A_1,A_2,A_3,C_1,C_2,C_3,A_4,A_5,A_6} \\ &\quad = |\psi\rangle_{A_1,A_2,A_3,C_1,C_2,C_3}|\mathrm{GHZ}\rangle_{A_4,A_5,A_6} \\ &\quad = \frac{1}{2\sqrt{2}}[|00,\alpha\rangle|00,\alpha\rangle|00,+\rangle \\ &\qquad + |01,\alpha\rangle|01,\alpha\rangle|01,+\rangle - |10,-\alpha\rangle|10,-\alpha\rangle|10,-\rangle \\ &\qquad - |11,-\alpha\rangle|11,-\alpha\rangle|11,-\rangle]_{A_1,A_4,C_2,A_3,A_6,C_3,A_2,A_5,C_1}. \end{aligned} \tag{18}$$

We now let atoms A_1 and A_3 fly through cavities C_2 and C_3, respectively, for time $t = \pi/2\lambda$ and then let atoms A_4 and A_6 fly through cavities C_2 and C_3, respectively, for time $t = \pi/2\lambda$. By doing this the state in modes A_1, A_4, C_2 evolves according to equations (9)–(11), giving

$$|00,\alpha\rangle_{A_1,A_4,C_2} \to \frac{1}{\sqrt{2}}[|B\rangle^{(1)}|\alpha\rangle + |B\rangle^{(2)}|-\alpha\rangle]_{A_1,A_4,C_2},$$

$$|01,\alpha\rangle_{A_1,A_4,C_2} \to \frac{1}{\sqrt{2}}[|B\rangle^{(3)}|\alpha\rangle + |B\rangle^{(4)}|-\alpha\rangle]_{A_1,A_4,C_2},$$

$$|10, -\alpha\rangle_{A_1,A_4,C_2} \rightarrow \frac{1}{\sqrt{2}}[|B\rangle^{(3)}|-\alpha\rangle - |B\rangle^{(4)}|\alpha\rangle]_{A_1,A_4,C_2},$$
$$|11, -\alpha\rangle_{A_1,A_4,C_2} \rightarrow \frac{1}{\sqrt{2}}[|B\rangle^{(1)}|-\alpha\rangle - |B\rangle^{(2)}|\alpha\rangle]_{A_1,A_4,C_2}. \quad (19)$$

Using equation (19) for modes A_1, A_4, C_2 and similar results for modes A_3, A_6, C_3 in equation (18), and then applying σ_z operation in mode A_2, the final output state is written as

$$\begin{aligned}|\psi\rangle_{A_1,A_4,A_3,A_6,A_2,A_5,C_1,C_2,C_3} = \frac{1}{2\sqrt{2}}[&|\lambda_I\rangle|\alpha, \alpha, \alpha\rangle \\ &+|\lambda_{II}\rangle|\alpha, \alpha, -\alpha\rangle + |\lambda_{III}\rangle|\alpha, -\alpha, \alpha\rangle \\ &+|\lambda_{IV}\rangle|\alpha, -\alpha, -\alpha\rangle + |\lambda_V\rangle|-\alpha, \alpha, \alpha\rangle \\ &+|\lambda_{VI}\rangle|-\alpha, \alpha, -\alpha\rangle + |\lambda_{VII}\rangle|-\alpha, -\alpha, \alpha\rangle \\ &+|\lambda_{VIII}\rangle|-\alpha, -\alpha, -\alpha\rangle]_{A_1,A_4,A_3,A_6,A_2,A_5,C_1,C_2,C_3}, \end{aligned} \quad (20)$$

where the first and second ket in each component on the RHS of equation (20) represent atomic states ordered in modes A_1, A_4, A_3, A_6, A_2, A_5 and cavity states ordered in modes C_1, C_2, C_3. States $|\lambda_i\rangle$ ($i = I, II, \ldots, VIII$) are orthonormalized six-qubit entangled states given in appendix A. From this appendix, it is clear that the entangled state $|\lambda_I\rangle$ is exactly the same as the particular SSE state used in section 2 for the SQIE protocol and all states are inter-convertible to each other just by applying local bit flip operation (σ_x) and phase flip operation (σ_z). So it is enough to generate any state $|\lambda_i\rangle$. For this, we now inject the coherent state $|\alpha\rangle$ in each cavity C_1, C_2, C_3, i.e. we again make use of the displacement operator $D(\alpha)$; doing so, the final output state in equation (20) becomes

$$\begin{aligned}|\psi\rangle_{A_1,A_4,A_3,A_6,A_2,A_5,C_1,C_2,C_3} = \frac{1}{2\sqrt{2}}[&|\lambda_I\rangle|2\alpha, 2\alpha, 2\alpha\rangle \\ &+|\lambda_{II}\rangle|2\alpha, 2\alpha, v\rangle + |\lambda_{III}\rangle|2\alpha, v, 2\alpha\rangle \\ &+|\lambda_{IV}\rangle|2\alpha, v, v\rangle + |\lambda_V\rangle|v, 2\alpha, 2\alpha\rangle \\ &+|\lambda_{VI}\rangle|v, 2\alpha, v\rangle + |\lambda_{VII}\rangle|v, v, 2\alpha\rangle \\ &+|\lambda_{VIII}\rangle|v, v, v\rangle]_{A_1,A_4,A_3,A_6,A_2,A_5,C_1,C_2,C_3}. \end{aligned} \quad (21)$$

Since $|2\alpha\rangle$ has a non-zero probability of having the vacuum state $|v\rangle$, for a clearer analysis, we expand the coherent state $|2\alpha\rangle$ into vacuum $|v\rangle$ and the state with non-zero numbers of photon $|Nz\rangle$,

$$|2\alpha\rangle = x\,|v\rangle + \sqrt{1 - x^2}\,|Nz\rangle\,, \quad (22)$$

where $x = e^{-2|\alpha|^2}$. Using equation (22) for $|2\alpha\rangle$ in equation (21), the final output state simplifies to

$$\begin{aligned}|\psi\rangle_{A_1,A_4,A_3,A_6,A_2,A_5,C_1,C_2,C_3} = \tfrac{1}{2\sqrt{2}}\big[&N_I^{-1}|\lambda_I'\rangle|Nz, Nz, Nz\rangle \\ &+N_{II}^{-1}|\lambda_{II}'\rangle|Nz, Nz, v\rangle + N_{III}^{-1}|\lambda_{III}'\rangle|Nz, v, Nz\rangle \\ &+N_{IV}^{-1}|\lambda_{IV}'\rangle|Nz, v, v\rangle + N_V^{-1}|\lambda_V'\rangle|v, Nz, Nz\rangle \\ &+N_{VI}^{-1}|\lambda_{VI}'\rangle|v, Nz, v\rangle + N_{VII}^{-1}|\lambda_{VII}'\rangle|v, v, Nz\rangle \\ &+N_{VIII}^{-1}|\lambda_{VIII}'\rangle|v, v, v\rangle\big]_{A_1,A_4,A_3,A_6,A_2,A_5,C_1,C_2,C_3}, \end{aligned} \quad (23)$$

where the normalized states $|\lambda_i'\rangle$ and factors N_i ($i = I, II, \ldots, VIII$) are given in appendix B. Now we perform photon-counting measurement (PCM) in cavity modes C_1, C_2, C_3. From equation (23), it is clear that there are eight possible PCM results: *I* (all modes give nonzero count), *II* (modes C_1 and C_2 give nonzero count and rest vacuum), *III* (modes C_1 and C_3 give nonzero count and rest vacuum), *IV* (mode C_1 gives nonzero count and rest vacuum), *V* (modes C_2 and C_3 give nonzero count and rest vacuum), *VI* (mode C_2 gives nonzero count and rest vacuum), *VII* (mode C_3 gives nonzero count and rest vacuum), *VIII* (all modes give vacuum). It is clear from equation (23) that the *i*th PCM result ($i = I, \ldots, VIII$) gives the state $|\lambda_i'\rangle$. From appendix B, we see that for $i = I$, the state $|\lambda_I'\rangle$ is the same as the required state $|\lambda_I\rangle$ (defined in appendix A and used for the SQIE protocol in section 2), while for $i = II, \ldots, VIII$, the generated state $|\lambda_i'\rangle$ is the superposition of the required state $|\lambda_i\rangle$ and the garbage state, e.g., for *II*, the PCM result generated state is $|\lambda_{II}'\rangle = N_{II}[(1-x^2)(x\,|\lambda_I\rangle + |\lambda_{II}\rangle)]$, so the $(1-x^2)x\,|\lambda_I\rangle$ part is the garbage state, while the required state is $|\lambda_{II}\rangle$. It is to be noted that for the appreciable mean photon number, $|\alpha|^2 \gg 0$, $x \ll 1$; therefore, a major part in $|\lambda_{II}'\rangle$ is of $|\lambda_{II}\rangle$.

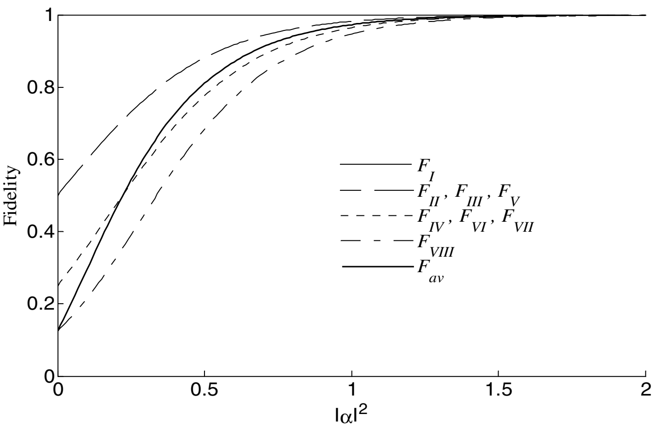

Figure 3. The variation of the fidelities of the generated state with respect to the required SSE state for each PCM result and the average fidelity with respect to the mean number of photons.

To estimate the quality of the generated state, we calculate the fidelity of $|\lambda_{II}'\rangle$ with respect to the required state $|\lambda_{II}\rangle$, which is defined as $F_i = |\langle\lambda_i'|\lambda_i\rangle|^2$, where $|\lambda_i\rangle$ is the required state, while $|\lambda_i'\rangle$ is actually the generated state. This gives $F_{II} = N_{II}^2(1 - x^2)^2 = (1 + x^2)^{-1}$. Similarly for all PCM results, $F_I = 1$, $F_{II} = F_{III} = F_V = (1 + x^2)^{-1}$, $F_{IV} = F_{VI} = F_{VII} = (1 + x^2)^{-2}$ and $F_{VIII} = (1 + x^2)^{-3}$. Figure 3 shows the variation of all these fidelities with respect to the mean number photon ($|\alpha|^2$), from where we see that F_I is always unity, while $F_{II,\ldots,VIII}$ become almost equal to unity for an appreciable mean number of photons, e.g., $F_{II} \geqslant 0.999$ for $|\alpha|^2 \geqslant 1.5$ and similar results for others. Hence all PCM results are expected to give the SSE state of high fidelity (almost perfect) for the appreciable mean number of photons. To estimate the overall quality of our scheme, we now calculate average fidelity defined as the summation of the products of the probability of occurrence and fidelity of the generated state for each PCM result. In our case, it is $F_{av} = \sum_{i=I}^{VIII} P_i F_i$, where $P_i = (1/8)\,|N_i|^{-2}$ and F_i are the probability of occurrence and corresponding fidelity of the generated state, respectively, for the *i*th PCM result. The average fidelity $F_{av} = (1/8)(2 - x^2)^3$ is shown in figure 3, from which it is clear that the average fidelity of the generated states with respect to the SSE state is almost unity, i.e. $F_{av} \geqslant 0.999$ for $|\alpha|^2 \geqslant 1.5$.