VALUATION OF VOLUME OF FRUSTUMIAL BODIES— A MATHEMATICAL STUDY

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Introduction:– The civil engineering profession has various a likeslike estimators, valuers, quantity surveyors and structural designers. All alikes have their own modus-operandi of doing their jobs for the satisfaction to the client. The work-styles vary with client to client and always does not remain same. Both find his own interest. Wherever it settles, it in finalized. With this pattern while I was teaching the subject i.e. Quantity, surveying & valuation, where above thought/ quotation set fit while assessing the volume of frustumial shaped bodies for various purposes like in soil embankment, concreting in column bases and road metalling. The criteria of assessing volume in all cases does not stand on similarity. These are assessed, conditional. As soil embanking is not considered a costly affair, so the calculation stress does not remain accurate, but while concreting and road metalling works are there, the same becomes costly & then precise assessment of volume of such shapes of bodies are done accurately.

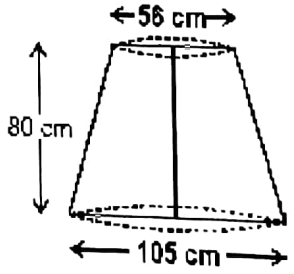
Here I think that we should have no concern with the condition, and always try to assess accurate and precise methods of calculating volume of furstumial bodies and should take into account only accurate procedure.

So far the methods are concerned which are publically in vogue always seen correct and all right but their results are varying. Here on which method we should trust/ it will be a method of assessing volume practically. This observation was done, while I was teaching the estimation of roads and concrete structures.

Here various methods, which are mathematically fit, have been discussed to assess the volume of same frustum-shape body for getting the true and real value of volume, which is the need of this tech paper, to make aware the professionals merged in calculation field.

Method 1:–To assess the volume, a frustum with its top & bottom dia. 56cm & 105cm and 80 cm in height has been considered as illustration before the readers. By the method–1, which may be considered as an Average Area Method for finding volume,

$$\text{Volume} = \left|\frac{A_1 + A_2}{2}\right| H$$



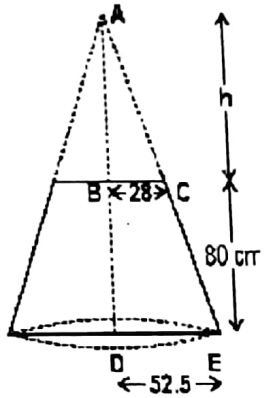
Where $A_1 = \frac{22}{7} \times \frac{56 \times 56}{2} = 2464 cm^2$

$$A_2 = \frac{22}{7} x \frac{105x105}{2} = 8662.5 \text{ cm}^2$$

H = 80 cm

$$\therefore \text{Volume } V_1 = \left|\frac{2464 + 8662.5}{2}\right| x80 = 445060 \text{ cm}^3$$

Method 2:– This method may be considered as a conical method



$$\Delta ABC \equiv \Delta ADE$$

$$\therefore \frac{AB}{AD} = \frac{BC}{DE}$$

$$\therefore \frac{h}{h+8} = \frac{28}{52.5}$$

$$\therefore h = 91.43\text{cm}$$

$$\therefore V_2 = \frac{1}{3}\left[\pi x52.5^2 x171.43 - \pi x28^2 x91.43\right]$$

$$= 419909.6 \text{ cm}^3$$

Method 3:– This method may be concluded as an Average Diameter Method

Here Dai = $\frac{56+105}{2} = 80.5\text{cm} = 80.5$ cm

$$\therefore V_3 = \frac{\pi}{4}\left[80.5\right]^2 x80 = 407330\text{cm}^3$$

Method 4:– $V_4 = \frac{\pi xH}{4}$ [$R^2 + r^2 + R \times r$].

$$= \frac{22}{7} x \frac{80}{3} \quad [52.5^2 + 28^2 + 52.5 \times 28]$$

$$= 419906.66 \text{ cm}^3$$

Method 5:– $V_5 = = \frac{H}{6}$ [$A_1 + A_2 + 4Am$]

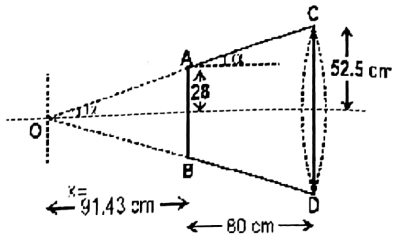
$$= \frac{80}{6}\left[\pi x52.5^2 x\pi x28^2 + 4x\pi\left[\frac{52.5+28}{2}\right]^2\right]$$

$$= 419906.66 \text{ cm}_3$$

Method 6:– $\tan\alpha = \frac{52.5-28}{80} = 0.30625$

$$\frac{28}{x} = \tan\alpha = 0.30625$$

$$\therefore x = 91.43\text{cm}$$



The equation of line OAC = y = x.tan α = 0.30625 x

$$\therefore V^6 = \int_{h_1}^{h_2} \pi y^2\, dx = \int_{91.43}^{171.43} \pi (0.30625)^2\, x^2\, dx = 419915 \text{ cm}^3$$

$$\therefore V_6 = 419915 \text{ cm}^3$$

Method 7:– Coordinate of Point A = (0, 28)

Coordinate of Point B = (80, 52.5)

Equation of line AB

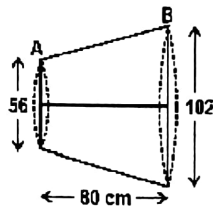
$$= y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$\rightarrow y - 28 = \left(\frac{52.5 - 28}{80 - 0}\right)(x - 0)$$

$$\therefore y = 0.306x + 28$$

$$\therefore y^2 = 0.0936x^2 + 17.136x + 784$$

$$\therefore \frac{dy}{dx} = 0.306$$



$$ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx = \sqrt{1+(0.306)^2}\cdot dx$$

$$\therefore ds = 1.046dx$$

$$V_7 = \int_0^{80} \pi y^2 ds = \left[\int_0^{80} (0.0936x^2 + 17.136x + 784) \times 1.046\, dx\right]\pi$$

$$= 431075.95\text{cm}^3$$

$$\therefore V_7 = 431075.95\text{cm}^3$$

Conclusion:- Various mathematical methods of determining the volume of frustum have been discussed which are applicable and prevailing in the market. A study has shown their differential aspect of resulting volumes. On practical and experience bases it has revealed that volume calculated through method number 4 & 5 are quite accurate. So the professionals merged in calculating field like in estimation, valuation in quantity surveying, preference must be given to real and true value methods shown through this paper.

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