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Unifying a Multitude of Common Fixed Point Theorems in Symmetric Spaces

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Abstract. The aim of this paper is to obtain some new common fixed point theorems for weakly compatible mappings in symmetric spaces satisfying an implicit function. Some illustrative examples to highlight the realized improvements are furnished. Our results generalize and extend some recent results contained in Ali and Imdad [Sarajevo J. Math. 4(17)(2008), 269-285] to symmetric spaces and consequently a host of metrical common fixed theorems are generalized and improved. We state an integral type fixed point theorem in symmetric space. In the process, we also derive a fixed point result on common fixed point in probabilistic symmetric spaces.

1. Introduction and Preliminaries

The celebrated Banach Contraction Principle is indeed the most fundamental result of metrical fixed point theory which is very effectively utilized to establish the existence of solutions of nonlinear Volterra integral equations, Fredholm integral equations, nonlinear integro-differential equations in Banach spaces besides supporting the convergence of algorithms in Computational Mathematics. However, sometimes one may come across situations wherein the full force of metric requirements are not used in the proofs of certain metrical fixed point theorems. Motivated by this fact, Hicks and Rhoades [16] proved some common fixed point theorems in symmetric spaces and showed that a general probabilistic structures admits a compatible symmetric or semi-metric. In 2006, Mihet [29] pointed out that Hicks and Rhoades [16] have inadvertently used a triangle inequality in their results.

Jungck [24] generalized the idea of weakly commuting pair of mappings due to Sessa [40] by introducing the notion of compatible mappings and showed that compatible pair of mappings commute on the set of coincidence points of the involved mappings. The study of common fixed points for non-compatible mappings is equally interesting due to Pant [32]. In 1996, Jungck [25] introduced the notion of weakly compatible mappings in non-metric spaces. For more details on systematic comparisons and illustrations of these described notions, we refer to Singh and Tomar [41] and Murthy [30]. The study of fixed points

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