Sample Paper-02
Mathematics
Class – XI

Answers

Section A

1. Solution

$$x = a + ib$$
$$|x| + x = \sqrt{a^2+b^2} + a + ib$$
$$\sqrt{a^2+b^2} + a = 2$$
$$a^2 + b^2 = (2-a)^2$$
$$b = 1$$
$$a^2 + 1 = 4 + a^2 - 4a$$
$$a = \frac{3}{4}$$
$$x = \frac{3}{4} + i$$

2. Solution

$$S = 1 + 3 + 5 + \cdots\cdots$$
$$S = \frac{n}{2}[2 + (n-1)(2)]$$
$$S = n^2$$

3. Solution

First term = 5

Sum of first and second term = 14

Second term= 9

Common Difference= $9 - 5 = 4$

n^{th} term = $5 + (n-1)4$

= $4n + 1$

4. Solution

Length of latus rectum of the ellipse = $\dfrac{2a^2}{b}$

Section B

5. Solution

$$f(x+5) = 5$$

6. Solution

The number of weights that can be measured = number of subsets can be formed excluding the null set

$2^4 - 1 = 15$

7. Solution

Let

$n = 1$

Then $n(n+1)(2n+1)$ = 6 and divisible by 6

Let it be divisible by 6 for

$n = m$

Then

$m(m+1)(2m+1) = 6k$ Where k is an integer

For $n = m+1$ the expression is

$$(m+1)(m+2)(2m+2+1) = (m+2)(m+1)(2m+1) + 2(m+1)(m+2)$$
$$= m(m+1)(2m+1) + 2(m+1)(2m+1) + 2(m+1)(m+2)$$
$$= m(m+1)(2m+1) + 2(m+1)(3m+3)$$
$$= m(m+1)(2m+1) + 6(m+1)^2$$
$$= 6k + 6(m+1)^2, This \quad is \quad divisible \quad by \quad 6$$

8. Solution

$1 - 2\sin^2 x - 5\sin x - 3 = 0$

$2sin^2 x + 5\sin x + 2 = 0$

Let $sinx = t$

Then, $2t^2 + 5t + 2 = 0$

Solving this quadratic

$2t(t+2) + (t+2) = 0$

$(2t+1)(t+2) = 0$

$t = -2, t = -\frac{1}{2}$

$\sin x = \frac{-1}{2}$

First value of t is rejected as $\sin x$ should lie between $(-1 \quad and \quad 1)$

General solution is $x = (-1)^{n+1}\frac{\pi}{6} + n\pi$

9. Solution

When

$m=0$

The given equation reduces to a first degree and it will have only one solution

Also when the discriminant is zero it will have only one solution

Discriminant is

$4(m+1)^2-4m^2.4=0$

$4(m^2+1+2m)-16m^2=0$

On simplifying and solving,

$(m-1)(3m+1)=0$

$m=1, m=-\frac{1}{3}$

Hence the three values of m for which the equation will have only one solution is

$m=0, m=1, m=-\frac{1}{3}$

10. Solution

$A.P \quad a-d, a.a+d$

$GP \quad \frac{b}{g}, b, bg$

$a-d+a+a+d=3a$

$3a=126$

$a=42$

$a+b=76$

$b=34$

$a-d+\frac{b}{g}=85\ldots(1)$

$a+d+bg=84\ldots(2)$

$2a+\frac{b}{g}+bg=169$

$34g^2-85g+34=0$

$g=\frac{85\pm\sqrt{85^2-4\times34\times34}}{2\times34}$

$g=2 \quad or \quad \frac{1}{2}$

$When \quad g=2$

$$42-d+\frac{34}{2}=85$$

$$d=-26$$

$$a=42,\quad d=-26,\quad g=2,\quad b=34$$

AP

68, 42, 16

GP

17, 34, 68

$$m=1, m=-\frac{1}{3}$$

11. Solution

$$f(x+1)=4^{x+1}$$

$$f(x)=4^{x}$$

$$f(x+1)-f(x)=4^{x+1}-4^{x}$$

$$=4^{x}.4-4^{x}$$

$$=4^{x}(3)$$

$$=3f(x)$$

12. Solution

$$\log\frac{1+\frac{3x+x^3}{1+3x^2}}{1-\frac{3x+x^3}{1+3x^2}}$$

$$=\log\frac{1+3x^2+3x+x^3}{1+3x^2-3x-x^3}$$

$$=\log\frac{(1+x)^3}{(1-x)^3}$$

$$=3\log\frac{(1+x)}{(1-x)}$$

$$=3f(x)$$

Section - C

13. Solution

$$\sin(45+30)=\sin 45\cos 30+\cos 45\sin 30$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\cos(45+30) = \cos 45 \cos 30 - \sin 45 \sin 30$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

14. Solution

$$\frac{sin3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = \frac{\sin 3\theta \cos\theta - \cos 3\theta \sin\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin(3\theta-\theta)}{\sin\theta\cos\theta}$$

$$= \frac{2\sin 2\theta}{2\sin\theta\cos\theta}$$

$$= \frac{2\sin 2\theta}{\sin 2\theta} = 2$$

15. Solution

$$x^2 + 4(mx+1)^2 = 1$$

$$x^2 + 4(m^2x^2 + 2mx + 1) = 1$$

$$x^2 + 4m^2x^2 + 8mx + 4 = 1$$

$$x^2(1+4m^2) + 8mx + 3 = 0$$

The line being a tangent ,it touches the ellipse at two coincident points, and so Discriminant must be zero,

$$(8m)^2 - 4(3)(1+4m^2) = 0$$

$$64m^2 - 12 - 48m^2 = 0$$

$$16m^2 = 12$$

$$m^2 = \frac{12}{16}$$

$$m^2 = \frac{3}{4}$$

16. Solution

Divide the equation by

$-\sqrt{3^2+-4^2}=-5$

Hence, $-\frac{3}{5}x+\frac{4}{5}y-4=0$

Where, $\cos\alpha=\frac{-3}{5}$ *and* $\sin\alpha=\frac{4}{5}$ *and* $p=4$

17. Solution

Multiply both numerator and denominator with $x-7$.Then denominator becomes a perfect square and it is always positive

Now

$(x+3)(x-7)\le 0$

Critical points are

$(-3,7)$

Hence, $-3\le x<7$

18. Solution

$$\lim_{x\to\infty}\frac{x^2-ax+4}{3x^2-bx+7}=\lim_{x\to\infty}\frac{x^2(1-\frac{a}{x}+\frac{4}{x^2})}{x^2(3-\frac{b}{x}+\frac{7}{x^2})}$$

$$=\frac{1}{3}$$

19. Solution

$$\lim_{x\to 0}\frac{\tan x}{\sin 3x}=\lim_{x\to 0}\frac{\sin x}{x}\times\frac{1}{cosx}\times\frac{1}{\frac{3sin3x}{3x}}$$

$$=1\times1\times\frac{1}{3}=\frac{1}{3}$$

20. Solution:

$y=\sin x$

$y+\Delta y=\sin(x+\Delta x)$

$\Delta y=\sin(x+\Delta x)-y$

$\Delta y=\sin(x+\Delta x)-\sin x$

$$\Delta y=2\cos\frac{2x+\Delta x}{2}\sin\frac{\Delta x}{2}$$

$$\frac{\Delta y}{\Delta x}=\frac{2\cos\frac{2x+\Delta x}{2}\sin\frac{\Delta x}{2}}{\Delta x}$$

$$\frac{\Delta y}{\Delta x}=\frac{\cos\frac{2x+\Delta x}{2}\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\cos x$$

$$\frac{dy}{dx}=\cos x$$

Note: As $\Delta x\to 0\,;\frac{\Delta x}{2}\ also\to 0$

21. Solution:

The successive First order of difference is $4,7,10,13,\ldots$ this is an AP.

The second order difference is(Difference of the first difference) $3,3,3,\ldots$

Third order difference (Difference of second order differences) is all 0

n th term

$$T_n=T_1+(n-1)\Delta T_1+\frac{(n-1)(n-2)}{2\,!}\Delta T_2+\frac{(n-1)(n-2)(n-3)}{3\,!}\Delta T_3$$

$$=\ 12+4(n-1)+3\frac{(n-1)(n-2)}{2}$$

$$=\frac{3n^2-n+22}{2}$$

Sum $=\ \frac{1}{2}\left(3\,\Sigma n^2-\Sigma n+22n\right)$

$$=\ \frac{1}{2}\left(3\,\frac{n(n+1)(2n+1)}{6}-\frac{n(n+1)}{2}+22n\right)$$

$$=\ \frac{1}{2}\left(n^3+n^2+22n\right)$$

22. Solution:

Let the point A be (x_1,y_1) and B be (x_2,y_2)

Let the point C be a point be (x, y) on the circle

Then AC and BC are perpendicular

Product of Slopes of line AC and BC =-1

$$\frac{y-y_1}{x-x_1}\cdot\frac{y-y_2}{x-x_2}=-1$$

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

23. Solution

x_i	f_i	$f_i x_i$	$\|x_i - 9\|$	$f_i \|x_i - 9\|$
5	14	70	4	56
7	6	42	2	12
9	2	18	0	0
10	2	20	1	2
12	2	24	3	6
15	4	60	6	24
	$N=\Sigma f_i=26$	$\Sigma f_i\ x_i=234$		$f_i\Sigma\|x_i-9\|=100$

$$Mean=\bar{X}=\frac{1}{N}(\Sigma f_i\ x_i)=\frac{234}{26}=9$$

$$MeanDeviation=M.D=\frac{1}{N}(\Sigma f_i\ |x_i-9|)=\frac{100}{26}=3.84$$

Section - D

24. Solution:

The odd digits $1,3,3,1$ can be arranged in their 4 places in $\frac{4!}{2!2!}$ ways

Even digits $2,4,2$ can be arranged in their 3 places in $\frac{3!}{2!}$

Hence the total number of arrangements = $\frac{4!}{2!2!}\times\frac{3!}{2!}=6\times3=18$ ways

25. Solution

Probability of one of them getting selected $P(E_1 or E_2)$ = 1- (Probability of both getting selected + Probability of none getting selected)

= $1-[P(E_1\cap E_2)+P(E_1'\cap E_2')]$

= $1-(\frac{1}{3}\times\frac{1}{5}+\frac{2}{3}\times\frac{4}{5})$

= $1-(\frac{1}{15}+\frac{8}{15})$

= $1-\frac{9}{15}=\frac{6}{15}=\frac{2}{5}$

26. Solution

Let A denote the set of numbers that are divisible by 2, B set of numbers that are divisible by 3, C set of numbers that are divisible by 5, D set of numbers that are divisible by both 2 and3 , E set of numbers that are divisible by both 2 and 5, F set of numbers that are divisible by 3 and 5, G set of numbers that are divisible by all the three numbers

$a+(n-1)d=T_n$

$n=\frac{T_n}{d}-\frac{a}{d}+1$

In this case $\frac{a}{d}=1$, Hence $n=integer\ part\ of\ \frac{T_n}{d}$

$n(A)=[\frac{1000}{2}]=500$

$n(B)=[\frac{1000}{3}]=333$

$n(C)=[\frac{1000}{5}]=200$

$n(D)=[\frac{1000}{2\times3}]=166$

$n(E)=[\frac{1000}{2\times5}]=100$

$n(F)=[\frac{1000}{3\times5}]=66$

$n(G)=[\frac{1000}{2\times3\times5}]=33$

Numbers that are divisible by 2, 3, 5 are

$n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cup B)-n(A\cup C)-n(B\cup C)+n(A\cap B\cap C)$

$=500+333+200+1666+100+66+33$

$=734$

Numbers that are not divisible by 2, 3, 5 are

$1000-734=266$